Master 1 EECS : High frequency electronics Exercises

1 Transmission line parameters

Consider the microstrip line in Figure 1. The considered substrate is Rogers RO4003, with a relative permittivity $\varepsilon_r = 3.38$, a substrate thickness h=787 µm, a loss tangent tan(δ)=0.0025. Copper conductors have a thickness t=18 µm, and a conductivity $\sigma = 58 \cdot 10^6$ S/m.

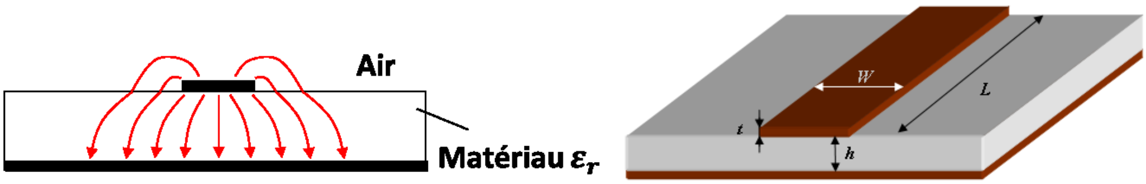

Figure 1: (a) Cross-sectional view and (b) 3-D view of a microstrip line

The EM field propagates partly in the air (above the conductor strip), partly in the substrate.

1.1 Characteristic impedance and equivalent effective permittivity

a) Give the expression of the characteristic impedance Z_C of the microstrip line as a function of the line parameters per unit length in the low-loss case.

b) The characteristic impedance of the microstrip line is shown in Figure 2(a). Explain the behaviour of Z_c as a function of the strip width W.

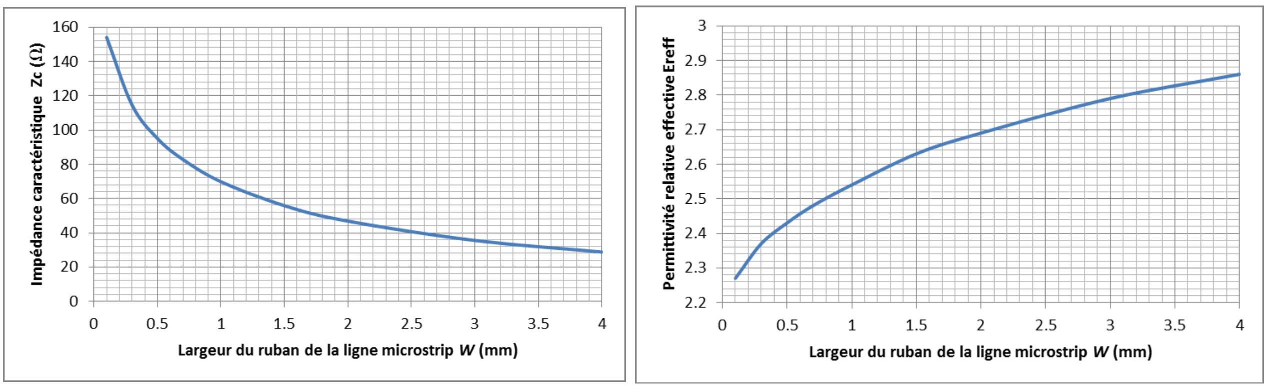

Figure 2: (a) characteristic impedance Z_c and effective relative permittivity ε_{reff} of the considered microstrip line line as a function of strip width W

c) Recall the expression of the propagation velocity v_φ as a function of ε_{reff}

d) Give the expression for the propagation velocity v_φ as a function of C_L and L_l

e) Explain what is the effective relative permittivity of a microstrip line?

f) The effective relative permittivity ε_{reff} of the transmission line is shown in Figure 2(b). Explain the behaviour ε_{reff} as a function of the strip width W.

A microstrip line with a characteristic impedance of 50 Ω is now required. The working frequency of is 2.45 GHz.

g) From Figure 2, deduce the width of the signal strip W.

h) From Figure 2, determine the value of the effective relative permittivity ε_{reff}.

i) Determine the value of v_φ.

1.2 *Electrical model RLCG*

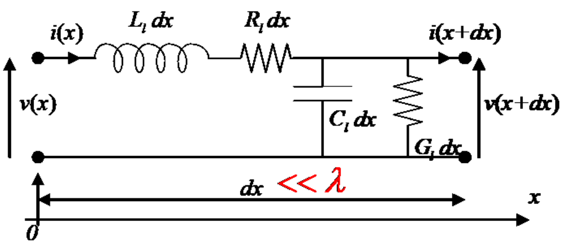

Figure 3: Electrical model of a transmission line

a) From the values of Z_c and v_φ determined above, deduce the values of the inductance per unit length L_l and the capacitance per unit length C_l.

b) Knowing C_l, deduce the value of the conductance per unit length G_l at 2.45 GHz. Let us indicate that the conductance is proportional to the capacitance C_l and to frequency. Moreover, the higher the loss tangent, $\tan(\delta)$, the higher G_l is. For a microstrip line, one can consider the following formula:

$$G_l = \omega \cdot C_l \cdot tan(\delta)$$

c) Calculate the resistance per unit length R_{l_0} at low frequency of the microstrip line. For that step, consider that this resistance is mainly due to the upper strip:

$$R_{l_0} = \rho \frac{1}{S} = \rho \frac{1}{W \cdot t}$$

where $\rho = 1/\sigma$ is the resistivity of the copper strip, $S = W \times t$ is the section in which the current flows.

d) Now, calculate the resistance per unit length R_l at the working frequency 2.45 GHz. For that, the skin effect of the metallic strips must be considered:

Thickness δ in which the current flows at high frequency

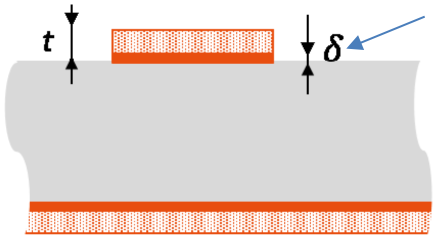

Figure 4: Cross-sectional view of a microstrip line. Highlighting of the "skin effect"

Skin effect:
At high frequencies, a non-uniform current distribution is observed in a conductor: decrease in current density when moving away from the periphery of the conductor → it induces a smaller section of the conductor in which the current flows.
Moreover, due to the proximity effect between the ground plane and the microstrip, the current flows over a thickness δ only at the interface between the strip and the substrate (as a first approximation).
As the frequency increases, the penetration thickness (skin thickness) δ of the current in the material is smaller:

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma}}$$

where $\mu_0 = 4\pi \cdot 10^{-7} H/m$, f is the frequency, and σ is the conductivity of the strip.

This reduction in the thickness of current flow leads to an increase in resistance (Joule effect losses). Hence, two cases occur:

- *if $\delta > t \rightarrow R_l = R_{l_0} = \rho \frac{1}{W \cdot t}$: in that case, since the skin depth is higher than the thickness of the strip t, the current flows in the total height of the strip.*
- *if $\delta < t \rightarrow R_l = \rho \frac{1}{W \cdot \delta}$: in that case, since the skin depth is smaller than the thickness of the strip t, the current flows in a smaller height δ. In that second case, since R_l is higher than R_{l_0}, losses will be increased.*

  Hence, after having calculated the skin depth δ at 2.45 GHz, calculate the resistance per unit length R_l.

1.3 Attenuation constant, phase constant

a) Determine the attenuation constant α by separating the conductive losses α_c and the dielectric losses α_d.

b) Deduce the attenuation loss $A_{dB/m}$ of the microstrip line in dB/m.

c) Determine the phase constant β of the microstrip line at 2.45 GHz.

d) A 50 Ω microstrip line of length $L = \lambda/2$ is expected. Determine this length in mm.

e) Deduce the attenuation A_{dB} between the input and the output of this microstrip line.

f) What power ratio does this correspond to?