M1 EECS

High Frequency Electronics

Transmission lines

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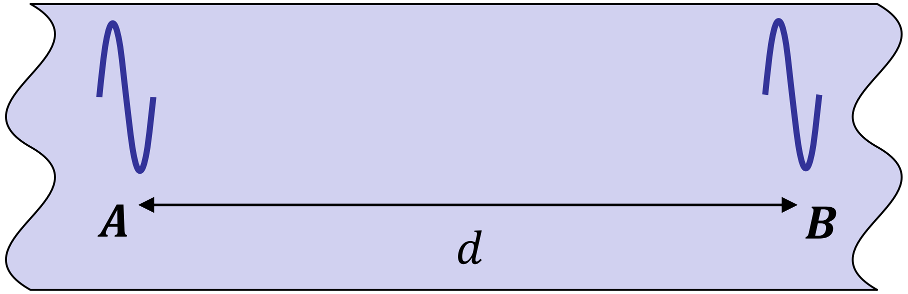
Propagation speed in an homogeneous medium

- Propagation velocity v_φ (m/s):

depends on the medium in which the electromagnetic (EM) field associated with the electrical signal propagates.



$\varepsilon_0 = 8{,}85 \times 10^{-12}$ F/m

$\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ H/m

- In vaccum, velocity = speed of light $\boxed{c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}} = 3 \cdot 10^8$ m/s

- In a medium of relative permittivity ε_r and relative permeability μ_r: $v_\varphi = \frac{c_0}{\sqrt{\varepsilon_r \cdot \mu_r}}$

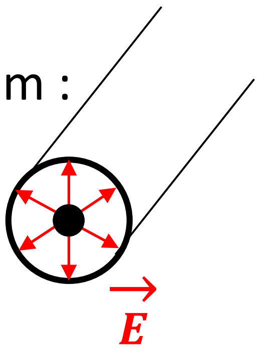
- In a non-magnetic material ($\mu_r = 1$) : $\boxed{v_\varphi = \frac{c_0}{\sqrt{\varepsilon_r}}}$

- Propagation time (s) : $\boxed{T_p = \frac{d}{v_\varphi}}$

d : distance travelled by the signal.

Propagation speed in a coaxial cable

- Homogeneous medium : the electromagnetic (EM) field is present in a single material
  - Example of a homogeneous medium :
    - Air
    - Coaxial cable : :



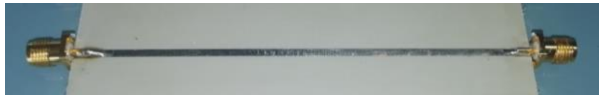
- 2 conductors
- 1 dielectric material ($\varepsilon_r,\ \mu_r$)

- Propagation velocity v_φ
  - Only one material "between" the "signal" conductor and the "ground" conductor

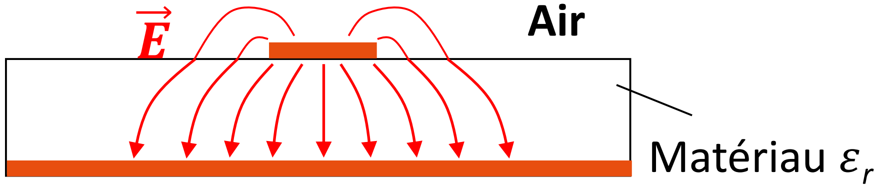
$$v_\varphi = \frac{c_0}{\sqrt{\varepsilon_r}}$$

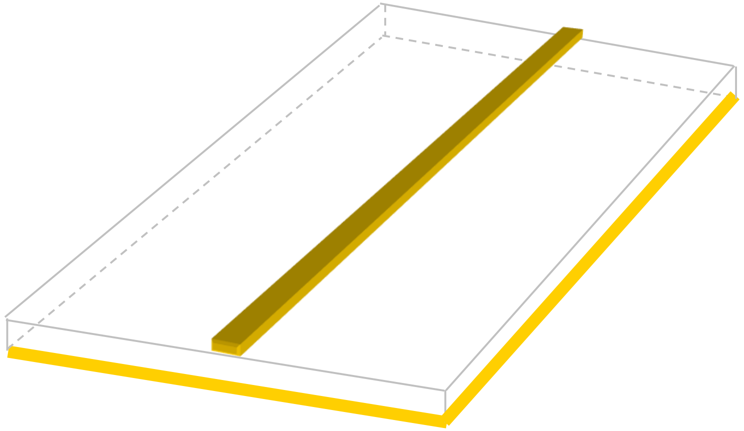
Propagation speed in an inhomogeneous medium

- Inhomogeneous medium: Electromagnetic field (EM) is present in several materials



  - Example: microstrip line (cross-sectional view)





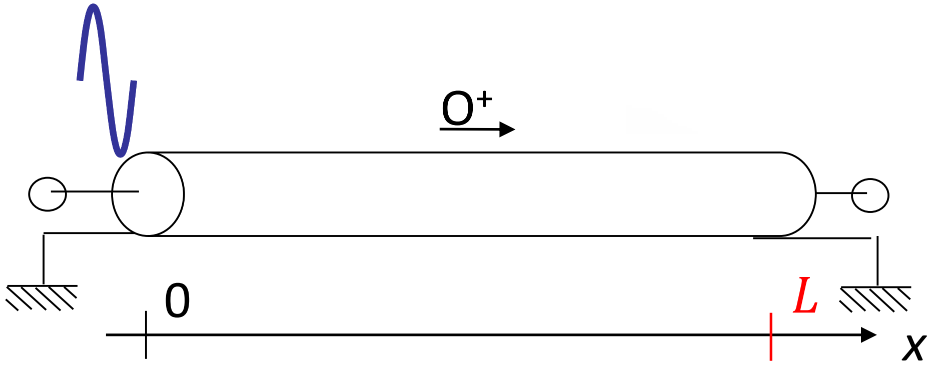
→ EM wave « sees » an effective medium with a effective relative permittivity ε_{reff}

- Propagation velocity v_φ in an inhomogeneous medium

$$\boxed{v_\varphi = \frac{c_0}{\sqrt{\varepsilon_{reff}}}}$$

Propagation constant

- Lossless transmission line:



V_{0+}

Travelling wave : $v^{+}(x;t) = V_M \cdot cos(\omega t - \beta x)$

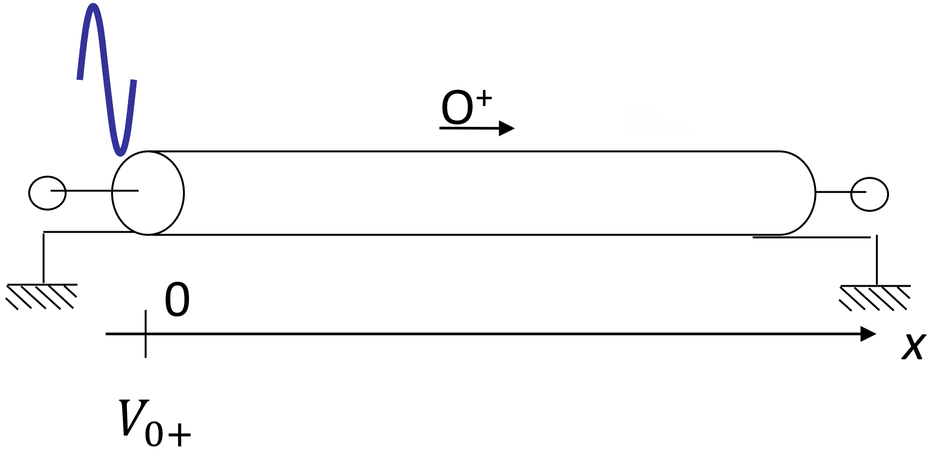
$V_M = V_{0+}$: constant

→ $\boldsymbol{\beta}$ **[rad/m] : phase constant** → $\boldsymbol{\beta = \frac{\omega}{v_\varphi}}$

$\boldsymbol{\theta = \beta \cdot L}$**: electrical length**

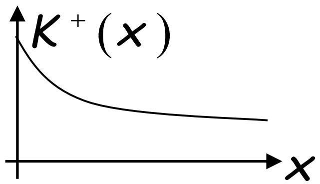
Propagation constant

- **Lossy** transmission line



Travelling wave : $v^{+}(x;t) = V_M \cdot cos(\omega t - \beta x)$ → $\boldsymbol{\beta}$ **[rad/m] : phase constant** → $\boldsymbol{\beta = \frac{\omega}{v_\varphi}}$

$V_M = V_{0+} \cdot K(x)$



$K(x) = e^{-\alpha \cdot x}$: expresses losses **when wave travels a distance x**

→ $\boldsymbol{\alpha}$ **[nepers/m] : attenuation constant** $(\boldsymbol{\alpha_{Np/m} > 0})$

→ $v^{+}(x,t) = V_{0+} \cdot e^{-\alpha \cdot x} \cdot cos(\omega t - \beta x)$

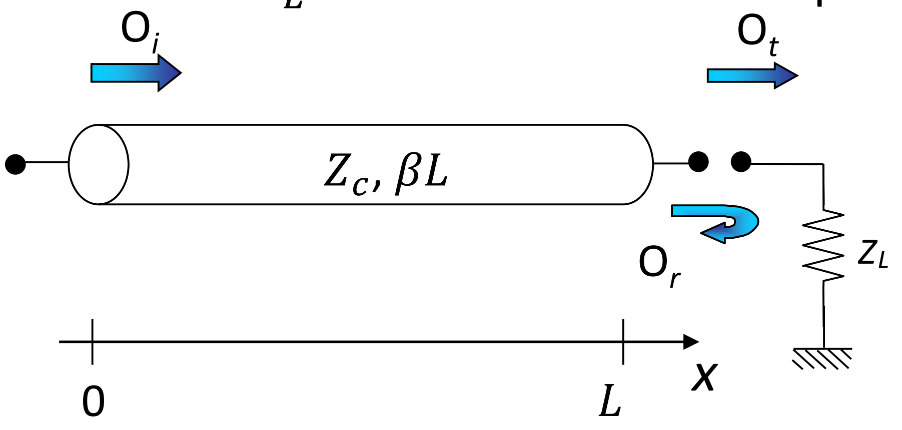
How to obtain the attenuation constant in dB/m? $\boldsymbol{\alpha_{dB/m} = 20 \cdot Log(e^{-\alpha}) = -8,68 \cdot \alpha_{np/m}}$

→ **Attenuation (dB) :** $\boldsymbol{A_{dB} = \alpha_{dB/m} \cdot x = -8,68 \cdot \alpha_{np/m} \cdot x}$ $(\boldsymbol{\alpha_{dB/m} < 0})$

Characteristic impedance

- What happens when a wave propagating in a line meets an interface/load ?
  - It depends on both : the load Z_L **&** the characteristic impedance Z_C of the transmission line

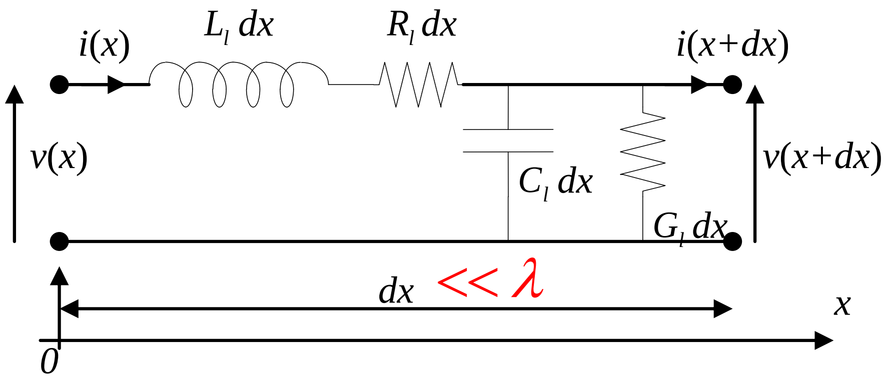
$$\Gamma = \frac{V_r}{V_i}$$



- Reflection coefficient $\Gamma = \frac{Z_L - Z_C}{Z_L + Z_C}$
  - In practice, the characteristic impedance Z_c of the Tline must be equal to the loaded impedance connected at the end of the Tline in order to suppress reflections.

Reflections mean that a part of the power is reflected ... and thus not transmitted!

Transmission line Electrical model



- Inductance per unit length L_l
- Capacitance per unit length C_l
- Resistance per unit length R_l
- Conductance per unit length G_l

- Low-loss transmission lines

Low-resistance conductors : Cuivre, Or, … → $\boldsymbol{R_l \ll j \cdot L_l\omega}$

Dielectric material with low-loss at mocrowave → $\boldsymbol{G_l \ll j \cdot C_l\omega}$

Phase constant β

$$\boxed{\beta = \omega \cdot \sqrt{L_l C_l}}$$

and $\boxed{v_\varphi = \frac{1}{\sqrt{L_l C_l}}}$

$(\beta = \omega / v_\varphi)$

Attenuation constant α:

$$\boxed{\alpha = \underbrace{\frac{1}{2} \cdot \frac{R_l}{Z_c}}_{\alpha_c} + \underbrace{\frac{1}{2} \cdot G_l \cdot Z_c}_{\alpha_d}}$$

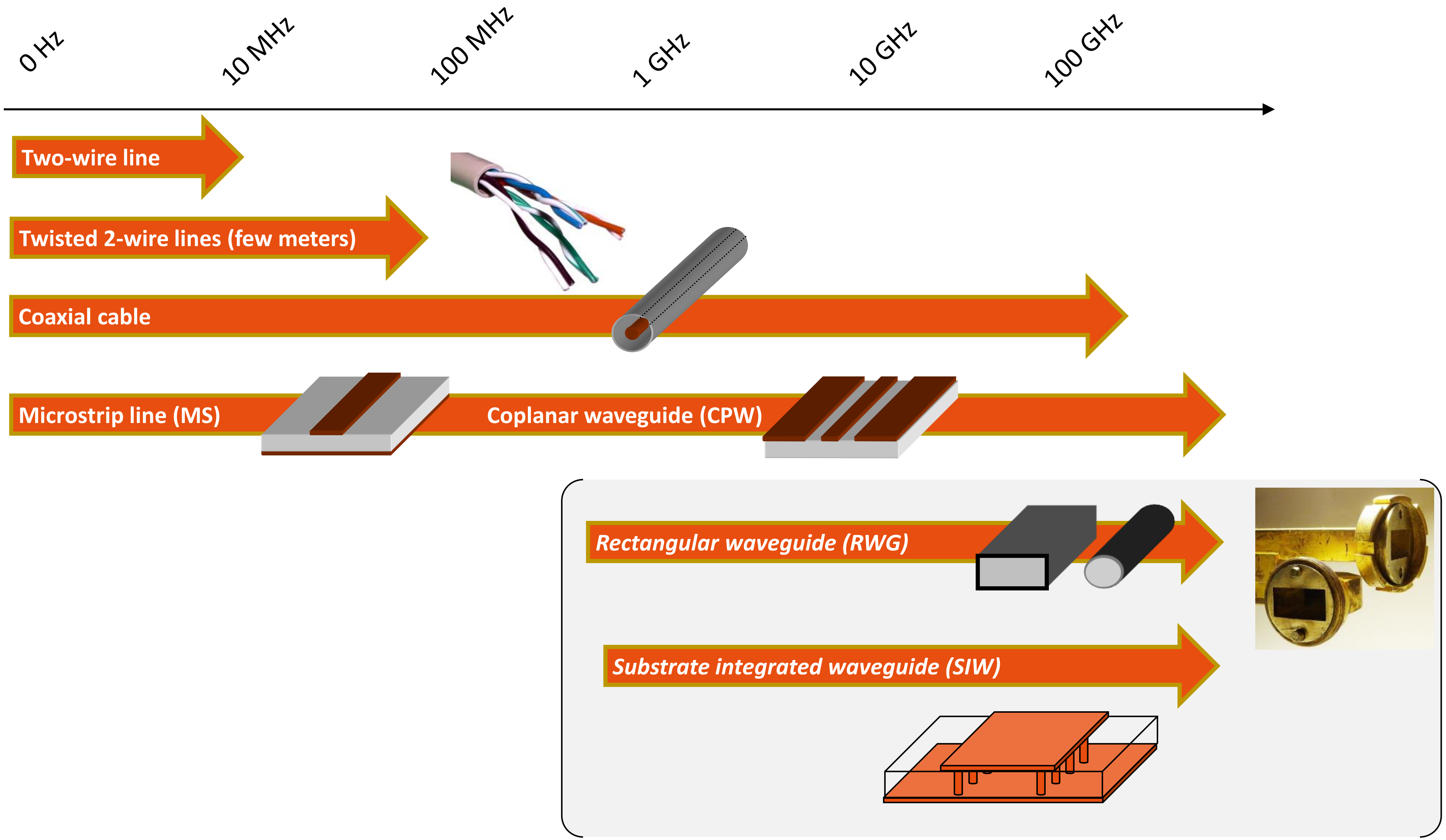
α_c : conductive loss

α_d : dielectric losses

Characteristic impedance Z_c

$$\boxed{Z_c = \sqrt{\frac{L_l}{C_l}}}$$

Examples of transmission lines



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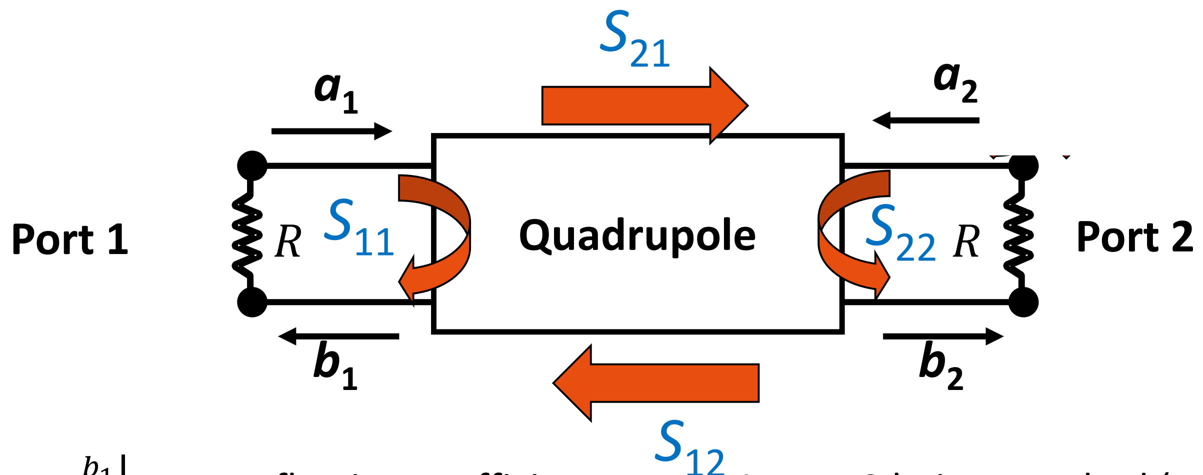
Frequency analysis

2022-2023

Scattering matrix : S matrix

- S parameters : allows a quadrupole to be described by means of waves



a_i : input wave (→ square root of the power injected at the access *i*)

b_j : output wave (→ square root of the power output from the access *j*)

$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$: reflection coefficient at port 1, port 2 being matched ($a_2 = 0$)
→ Return loss at port 1

$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$: transmission coefficient from port 1 to port 2, port 2 being matched ($a_2 = 0$)
→ Insertion loss from port 1 to port 2

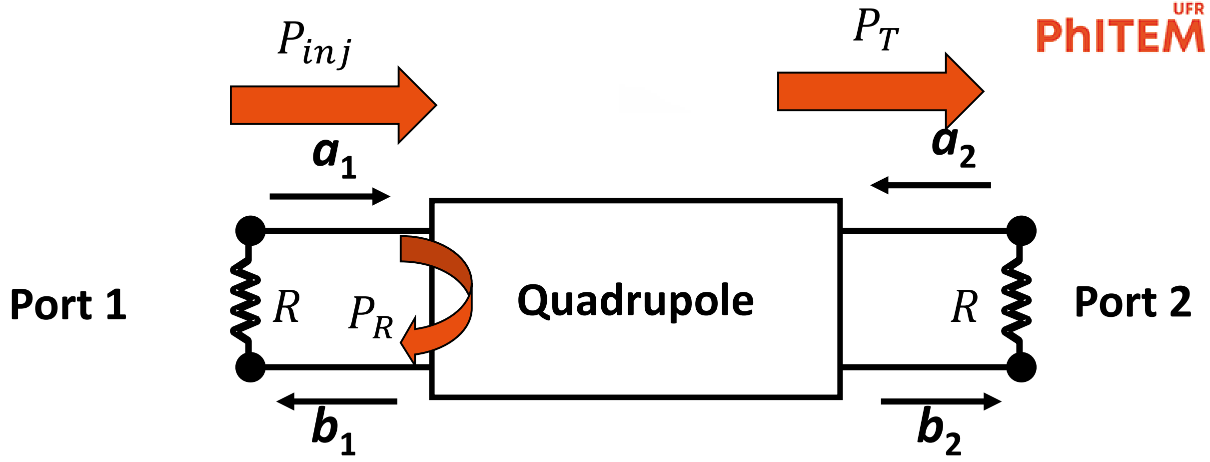
$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$: reflection coefficient at port 2, port 1 being matched ($a_1 = 0$)
→ Return loss at port 2

$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$: transmission coefficient from port 2 to port 1, port 1 being matched ($a_1 = 0$)
→ Insertion loss from port 2 to port 1

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2$$
$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Scattering matrix : S matrix



- Properties :
  - Symetric quadrupole → S_{11}=S_{22}
  - Reciprocal quadrupole → S_{21} = S_{12}
  - Lossless circuit:
    - The total power leaving the N ports is equal to the total power entering. Therefore, the total power injected at the input of the circuit must be equal to the sum of the outgoing power at the output and the reflected power at the input:

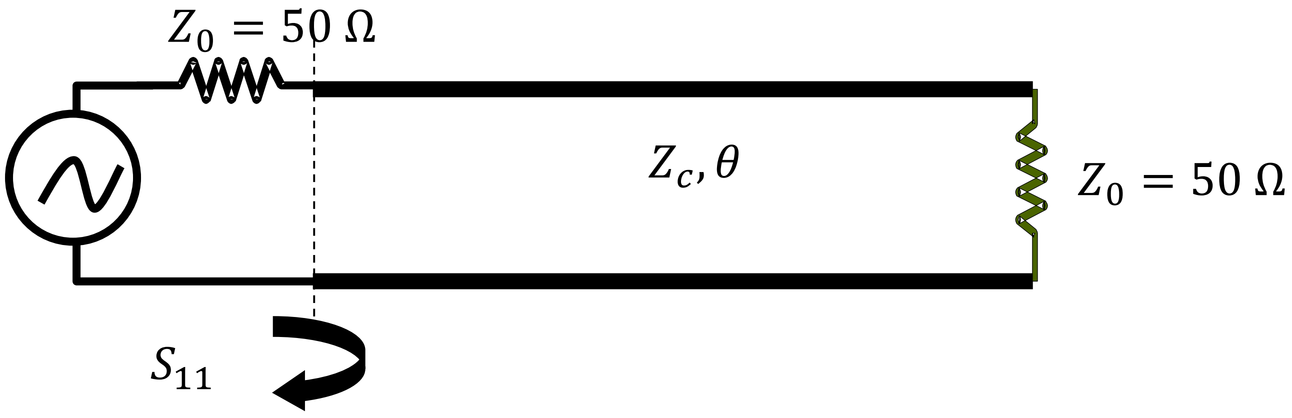
$$P_R + P_T = P_{inj} \quad \Rightarrow \frac{P_R}{P_{inj}} + \frac{P_T}{P_{inj}} = 1 \quad \Rightarrow \quad |S_{11}|^2 + |S_{21}|^2 = 1$$

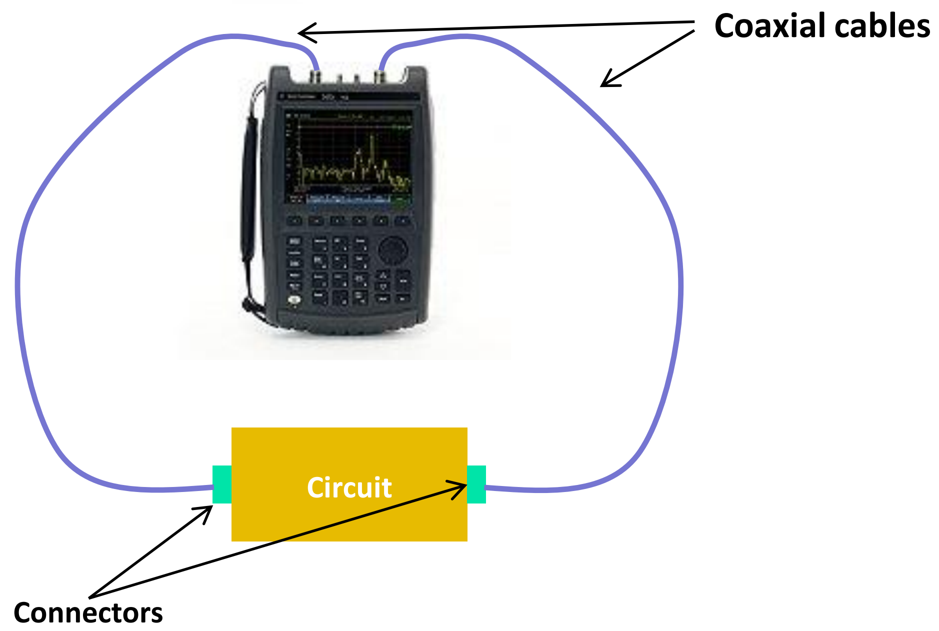
  - Case of a lossy circuit:
    - The sum of the transmitted and reflected power is not equal to the incident power. The circuit is lossy! The difference represents the losses of the circuit, i.e. the power dissipated by it.
    - The losses can be deduced from the S parameters:

$$\frac{P_{diss}}{P_{inj}} = 1 - (|S_{11}|^2 + |S_{21}|^2)$$

S Parameters measurements of a transmission line

- Return loss → matching





- For a lossless Tline, with a characteristic impedance Z_c and an electrical length $\theta = \beta L$

$$S_{11} = \frac{j \cdot tan(\theta) \cdot (Z_c^2 - Z_0^2)}{2 Z_c Z_0 + j \cdot tan(\theta) \cdot (Z_c^2 + Z_0^2)}$$

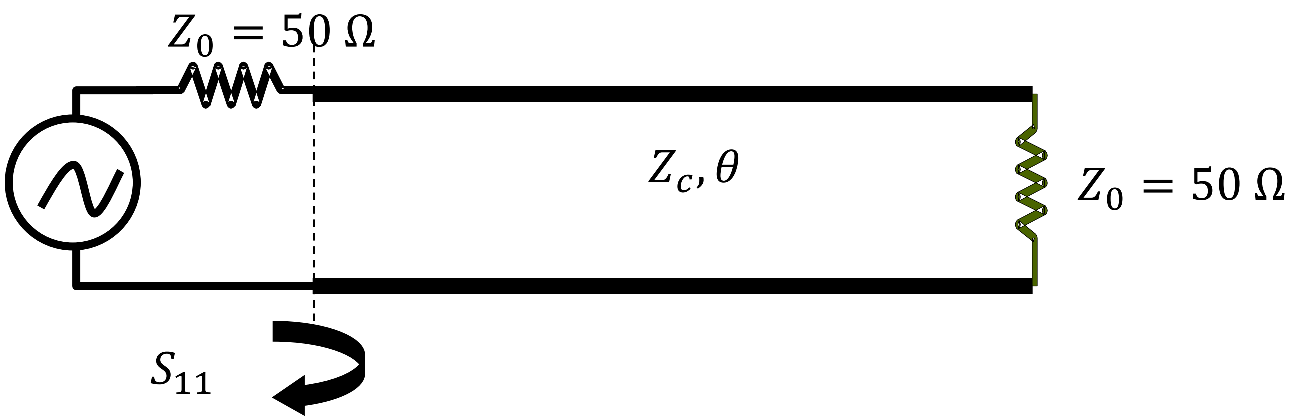
To be demonstrated for the labwork

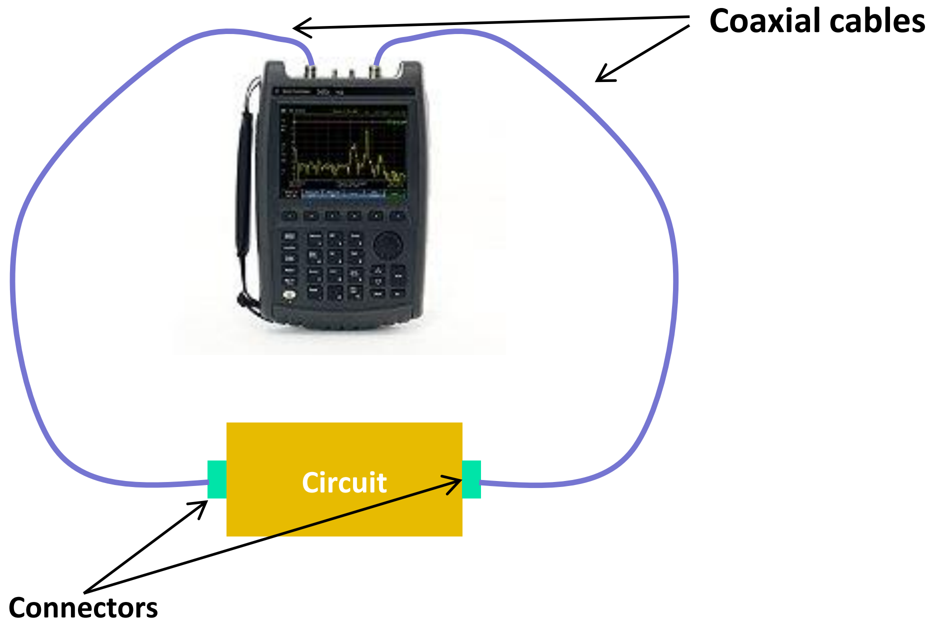
- If $Z_c = Z_0$ → $S_{11} =$

If $S_{11} = 0$ → $|S_{21}|$

S Parameters measurements of a transmission line

- Return loss → matching





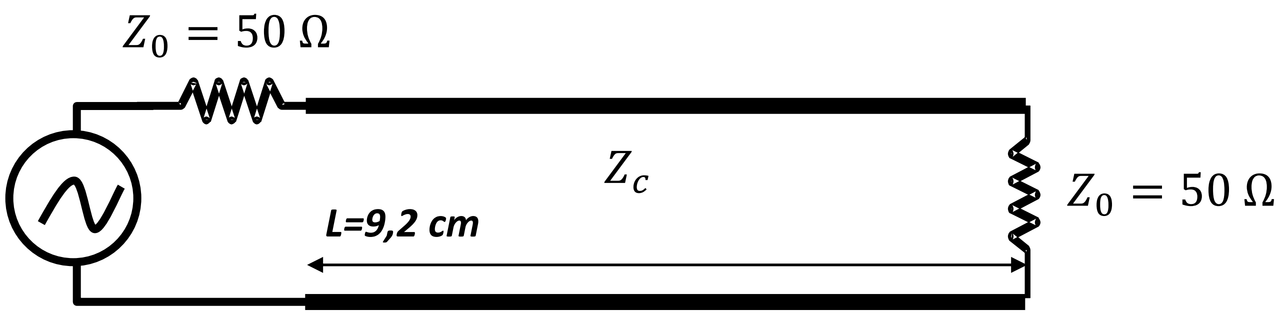
- For a lossless Tline, with a characteristic impedance Z_c and an electrical length $\theta = \beta L$

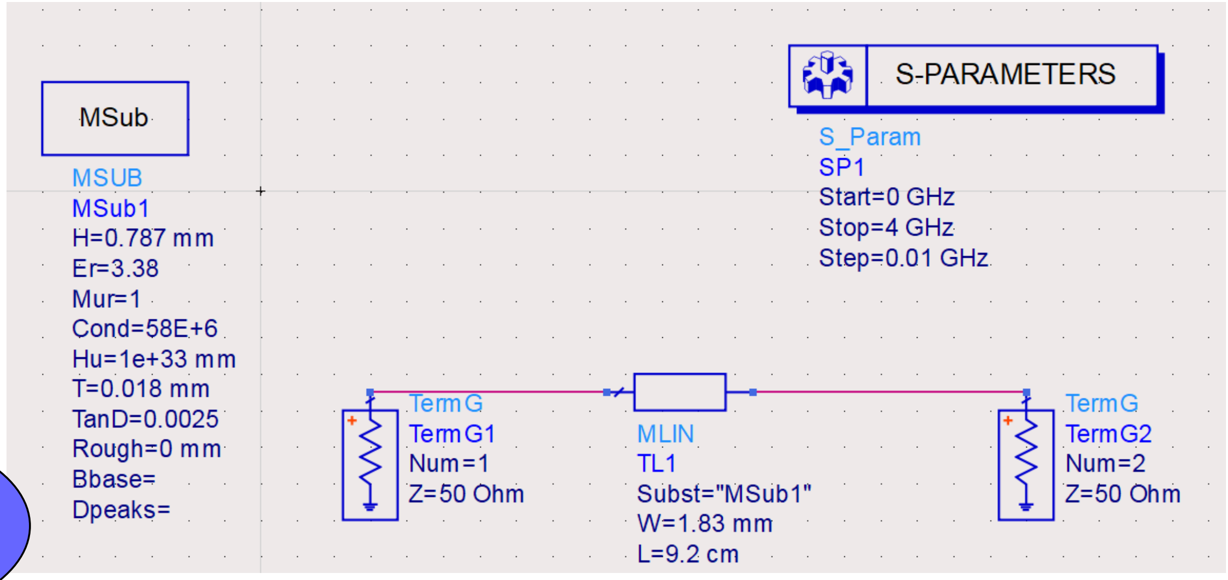
$$S_{11} = \frac{j \cdot tan(\theta) \cdot (Z_c^2 - Z_0^2)}{2Z_cZ_0 + j \cdot tan(\theta) \cdot (Z_c^2 + Z_0^2)}$$

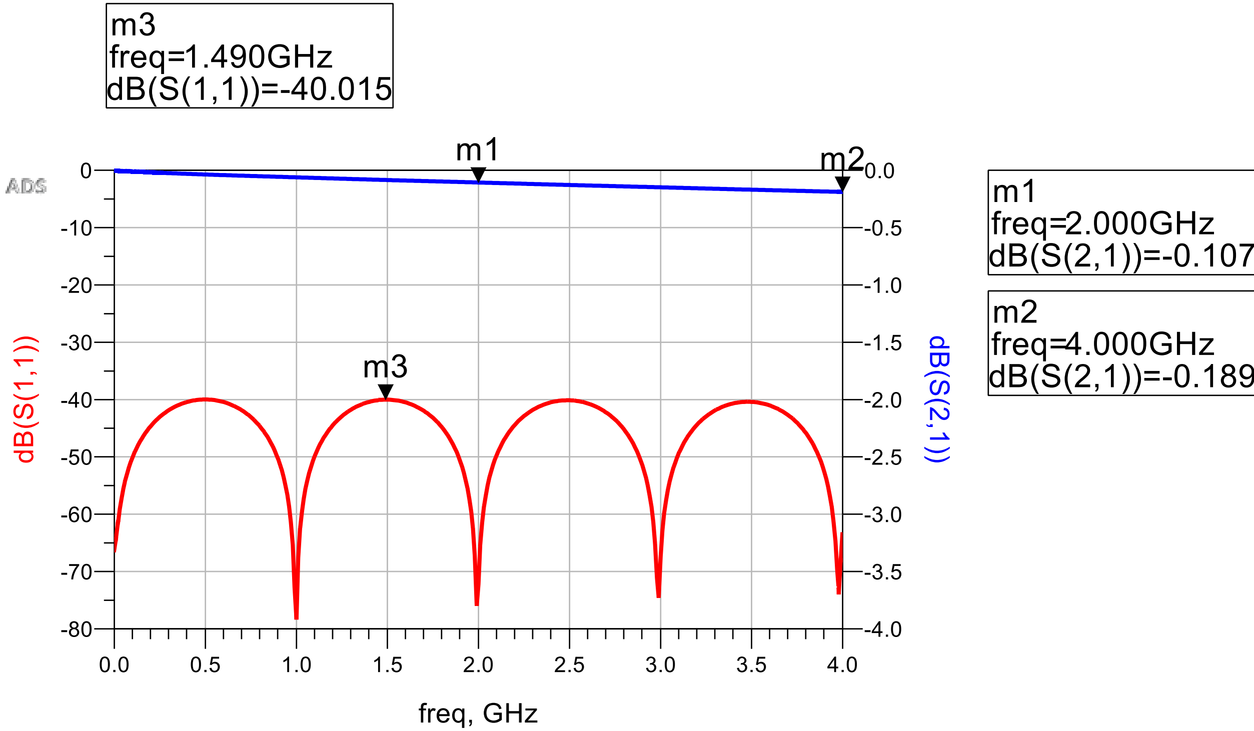
- If $Z_c \neq Z_0$ → $S_{11} = ?$
  - If $\theta = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$
  - If $\theta = 0, \pi, 2\pi \ldots$

S parameters of a transmission line: Example

- Tline of length L=9.2cm and of unknown Z_c





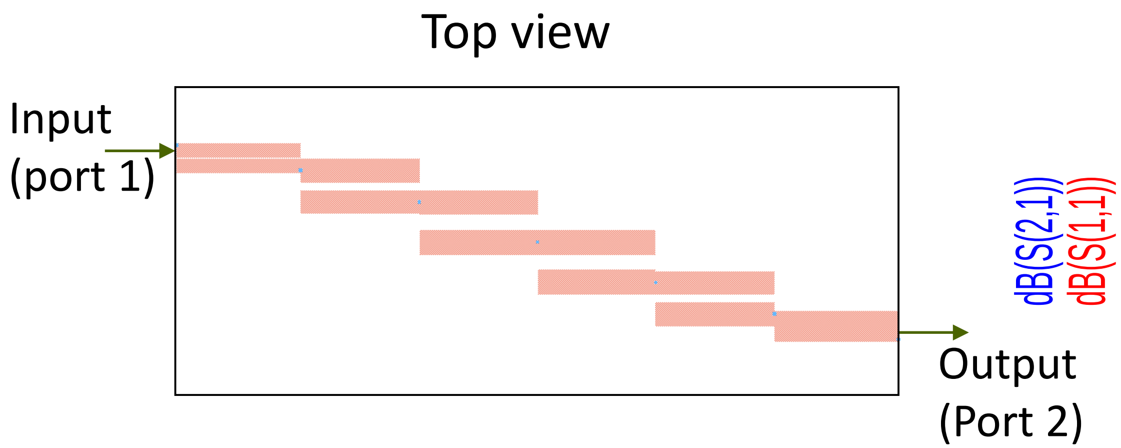


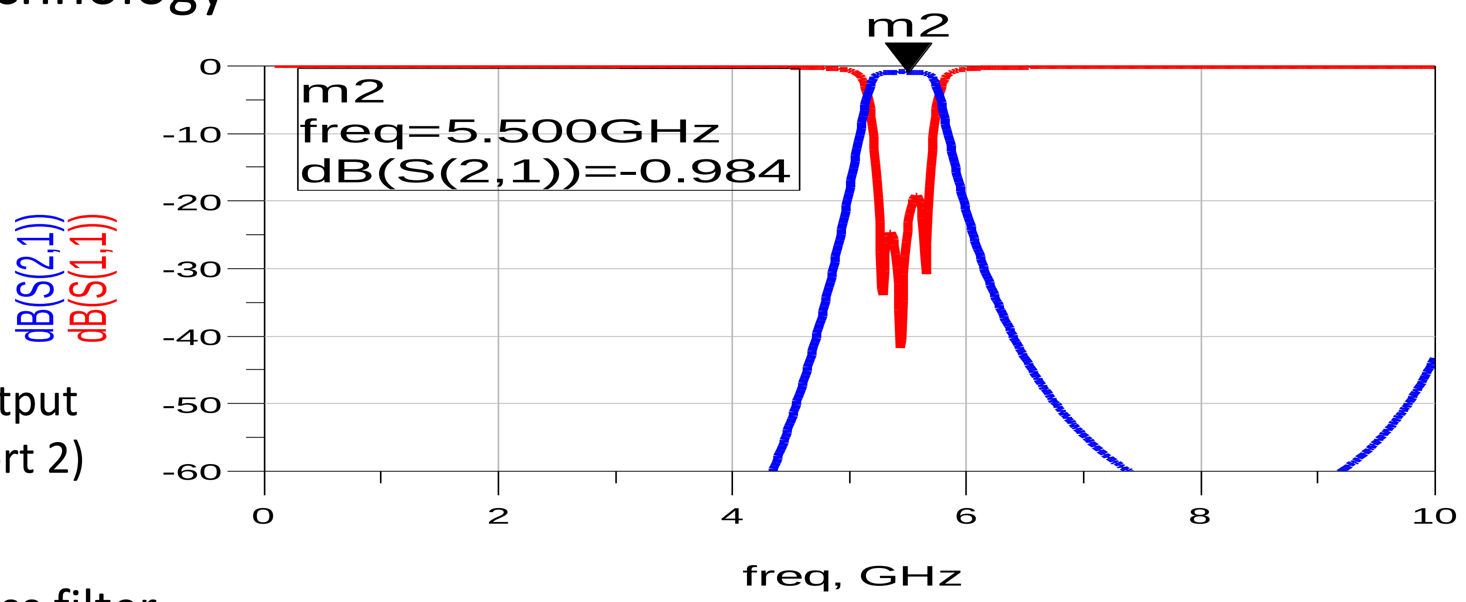
Insertion loss S_{21} and return loss S_{11}

- Why $|S_{11}| = |S_{22}|$? Why $|S_{21}| = |S_{12}|$?
- Without calculus, indicate an approximative value of Z_c
- What % of the total power is dissipated by the Tline at 2 GHz and at 4 GHz?
- Calculate the attenuation constant at 2 GHz.

S parameters of a microwave filter

- Passband filter in microstrip technology



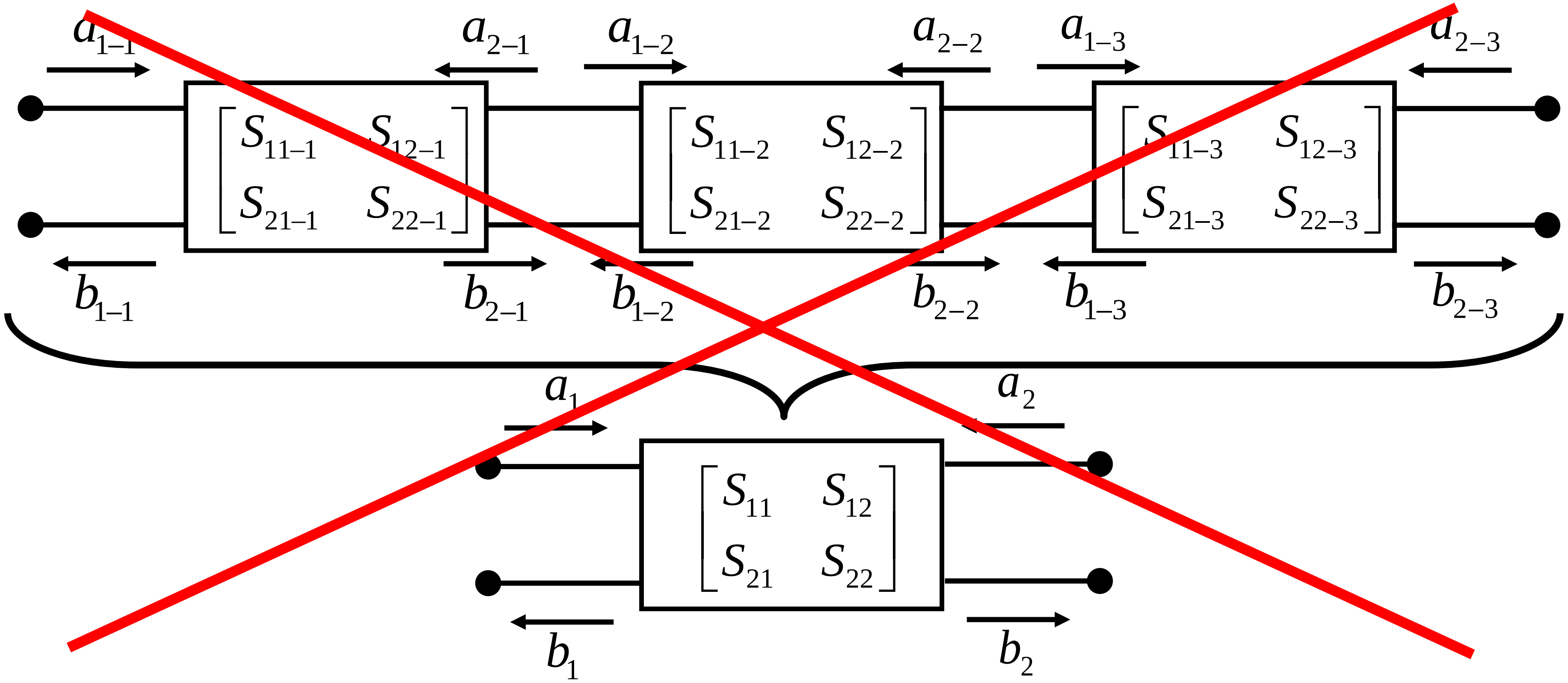


- Justify the fact that it is a bandpass filter
- What is the insertion loss at center frequency?
- What is the return loss at center frequency?
- What % of the total power is transmitted through the filter at the center frequency?
- What % of the total power is reflected at the input of the filter at the center frequency?
- What % of the total power is dissipated by the filter at the center frequency?

How to obtain the scattering matrix of a system constituted of several circuits?

- Cascaded circuits

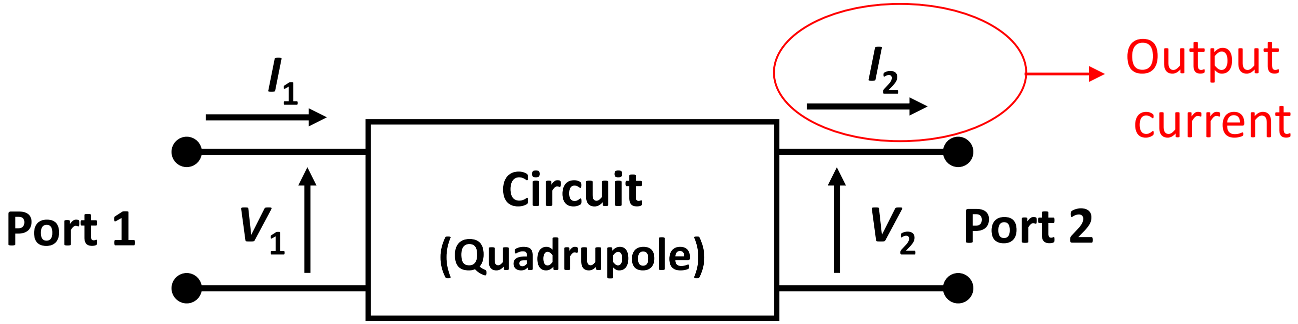
Scattering matrix: $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = f\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right)$



It is incorrect to multiply the S-matrices of each circuit to obtain the S-matrix of the complete system.

Need to consider another mathematical tool: Cascaded Matrix → ABCD matrix

- ABCD matrix: allows describing a quadrupole using currents and voltages

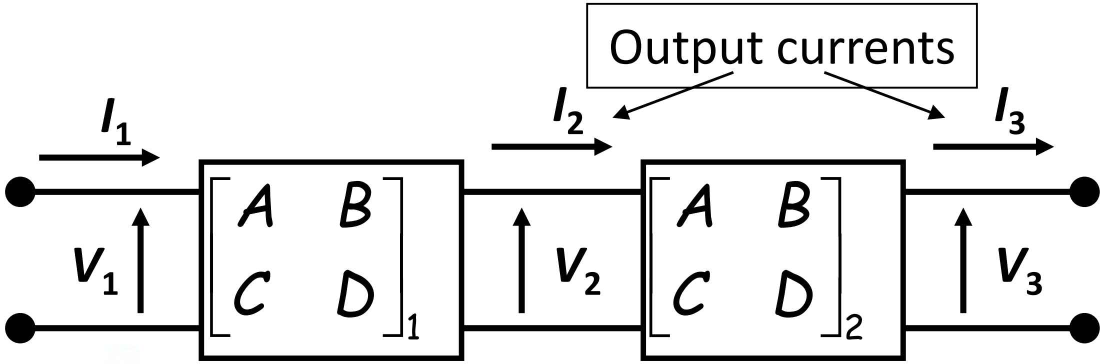


$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$\begin{vmatrix} V_1 \\ I_1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \cdot \begin{vmatrix} V_2 \\ I_2 \end{vmatrix}$$

$$A = \left|\frac{V_1}{V_2}\right|_{I_2=0},\ B = \left|\frac{V_1}{I_2}\right|_{V_2=0},\ C = \left|\frac{I_1}{V_2}\right|_{I_2=0},\ D = \left|\frac{I_1}{I_2}\right|_{V_2=0}$$

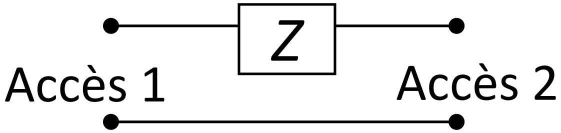
- ABCD matrix → suitable for cascading of circuits



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

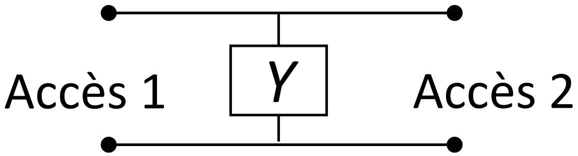
ABCD Matrix of lumped circuits

- Series Impedance



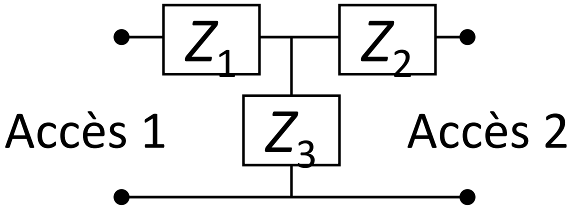
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

- Shunt admittance



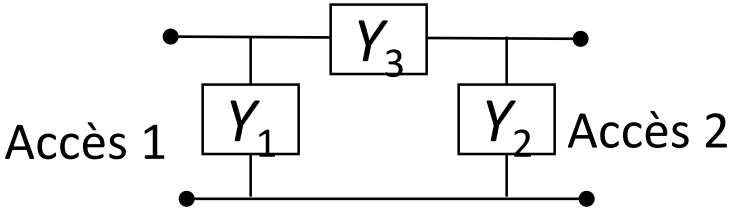
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

- T cell



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + Z_1/Z_3 & Z_1 + Z_2 + Z_1 Z_2/Z_3 \\ 1/Z_3 & 1 + Z_2/Z_3 \end{bmatrix}$$

- π cell

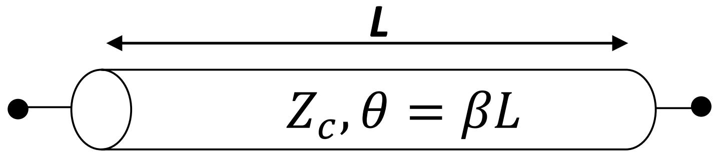


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + Y_2/Y_3 & 1/Y_3 \\ Y_1 + Y_2 + Y_1 Y_2/Y_3 & 1 + Y_1/Y_3 \end{bmatrix}$$

ABCD Matrix of a lossless transmission line

- Lossless transmission line

Port 1



Port 2

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\theta) & j \cdot Z_c \cdot \sin(\theta) \\ j \cdot \frac{\sin(\theta)}{Z_c} & \cos(\theta) \end{bmatrix}$$

- Exercise : Calculate the input impedance Z_{in} of the Tline loaded by an impedance Z_L

Pros and Cons of ABCD Matrix

☺ **Advantages:**

- Very intuitive
- Describes each access by current voltage
- Simple calculation for cascaded systems

☹ **Disadvantages:**

- Difficult to measure → need to consider another matrix: the more easily measurable S-matrix!
- Matrix ABCD deduced from the measurement by conversion of the measured S-matrix

Conversion formulas: [ABCD]→[S] and [S]→[ABCD]

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$$

$$B = Z_o \frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$$

$$C = \frac{1}{Z_o} \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$$

$$D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$$

$$S_{11} = \frac{A+B/Z_o-CZ_o-D}{A+B/Z_o+CZ_o+D}$$

$$S_{12} = \frac{2(AD-BC)}{A+B/Z_o+CZ_o+D}$$

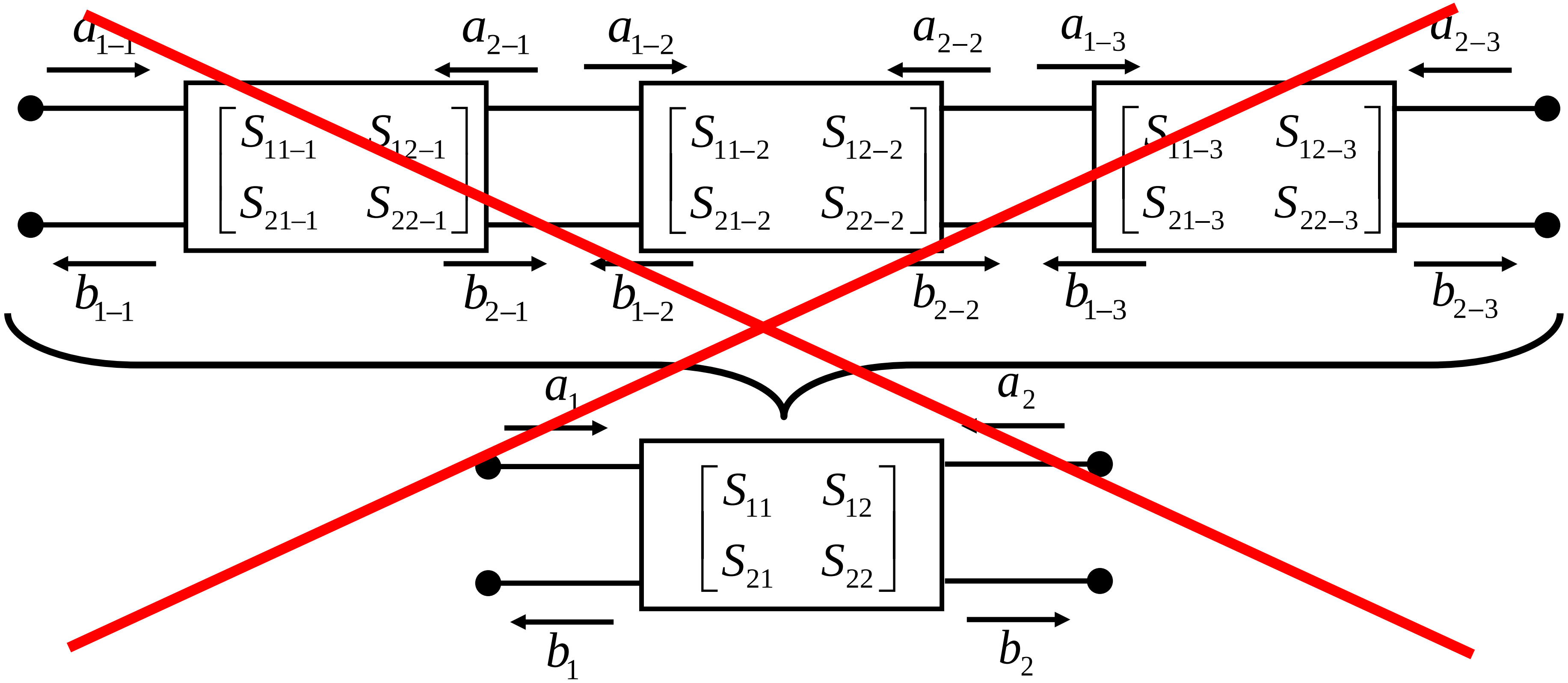
$$S_{21} = \frac{2}{A+B/Z_o+CZ_o+D}$$

$$S_{22} = \frac{-A+B/Z_o-CZ_o+D}{A+B/Z_o+CZ_o+D}$$

How to obtain the scattering matrix of a system constituted of several circuits?

- Cascaded circuits

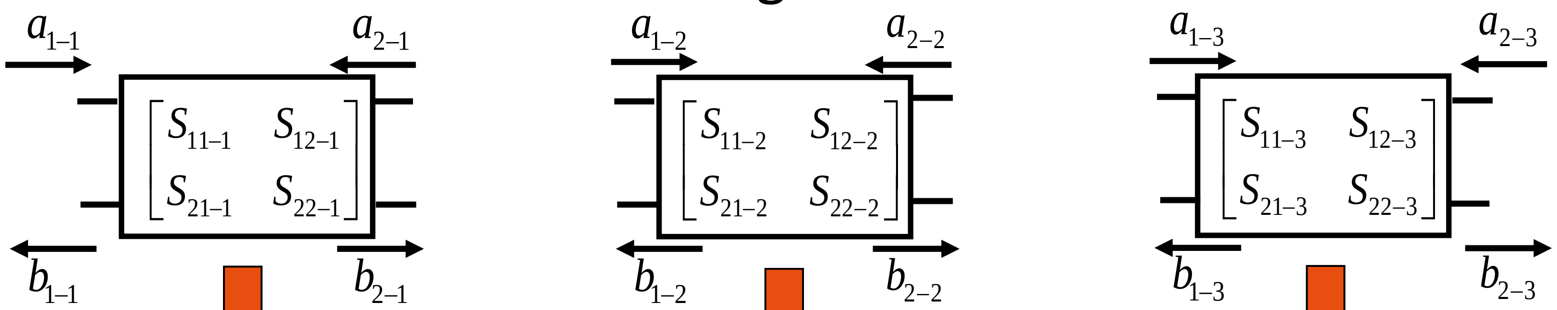
Scattering matrix: $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = f\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right)$



It is incorrect to multiply the S-matrices of each circuit to obtain the S-matrix of the complete system.

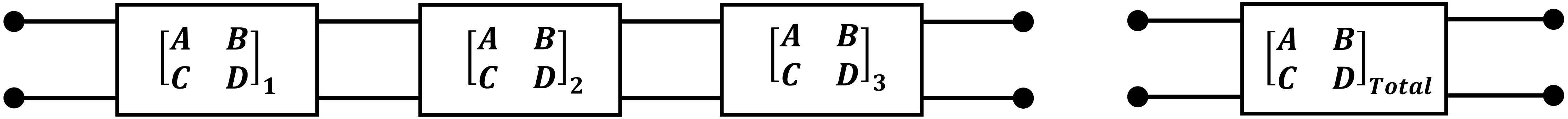
How to obtain the scattering matrix of a system constituted of several circuits?

- How can we obtain the resulting S matrix of three circuits whose S matrices are known?

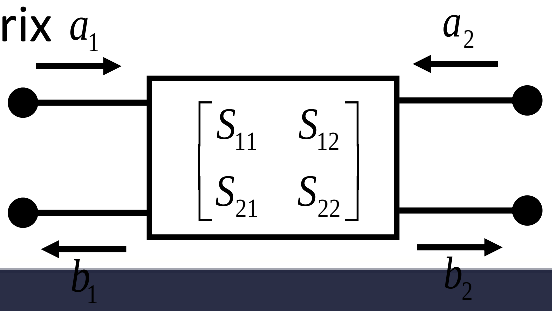


- We convert the S-matrices of each circuit into an ABCD-matrix; then we calculate the total ABCD-matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \times \begin{bmatrix} A & B \\ C & D \end{bmatrix}_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Total}$$



- Convert the total ABCD matrix to obtain the total S matrix



$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{Total}$$

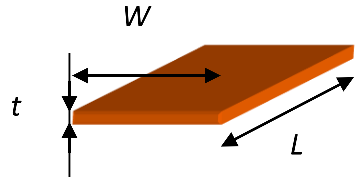
Appendix 1

Tline Electrical model

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line

- Resistance per unit length R_l :
  - 1 - Resistance of the metallic strip of width W, thickness t and length L



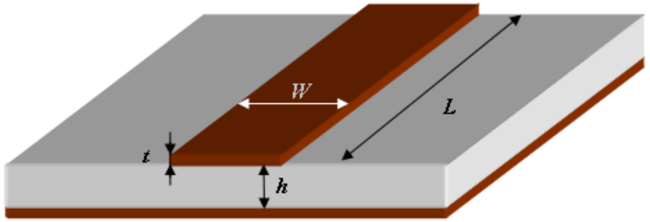
$$R = \rho \frac{L}{W \cdot t}$$

ρ = 1/ σ: resistivity of the conductor (in Ω.m)

Résistance per unit length $$R_{l_0} = \rho \frac{1}{W \cdot t}$$

  - 2 – Resistance per unit length of the microstrip line:

Must consider the upper strip and the down metallic ground plane



$$\Rightarrow R_l = R_{l_{upper\ strip}} + R_{l_{ground\ plane}}$$

In practice, since the ground plane is much larger than the upper strip

$$\Rightarrow R_l \approx R_{l_{ruban\ signal}}$$

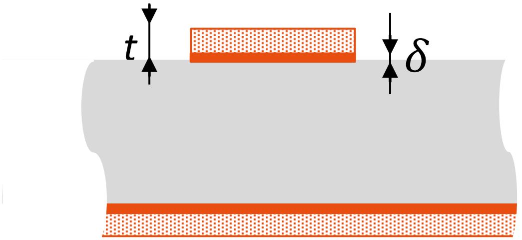
  - /!\ : the resistance increases at high frequency due to skin effect…

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line

- Resistance per unit length R_l when considerinf the skin effect:
  - 3 – Skin effect:
    - At high frequencies, a non-uniform current distribution is observed in a conductor: inducing a decrease in current density when moving away from the periphery of the conductor → it induces a smaller section of the conductor in which the current flows.
    - Moreover, due to the proximity effect between the ground plane and the microstrip, the current flows over a thickness δ only at the interface between the strip and the substrate (as a first approximation).
    - As the frequency increases, the penetration thickness (skin thickness) δ of the current in the material is smaller:

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu \cdot \sigma}}$$



Cross-sectional view of a microstrip line.
Highlighting of the "skin effect"

  - Decrease of the *thickness in which the current flows* → *increase of losses due to Joule effect.*

Si $\delta < t \rightarrow \quad R_l = \rho \frac{1}{W \cdot \delta}$

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line

$$\mu_0 = 4\pi \cdot 10^{-7} \mathrm{H/m}$$
$$\varepsilon_0 = 8.85 \cdot 10^{-12} F/\mathrm{m}$$

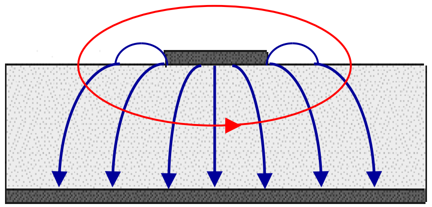
- Exercise : Consider a metallic strip of width W=1,8 mm with a metallization thickness of 35µm. Consider that the ground plane has an infinite width.
  - Calculate the skin depth δ at 50 Hz and 1 GHz.
  - Calculate the conduction surface.
  - Calculate the resistance per unit length of the microstrip line

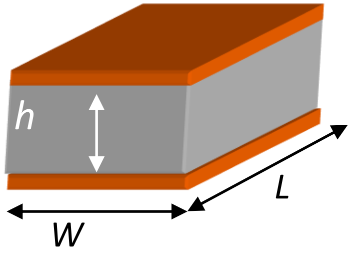
Frequency (Hz)	Skin depth (mm)	Conduction surface (mm²)	Resistance per unit length (Ω/m)
50			
1 kHz	2.09	0.063	0.274
1 MHz	0.066	0.063	0.274
1 GHz			
10 GHz	0.00066	0.0012	14.5
100 GHz	0.00021	0.00038	45.8

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line

- Capacitor per unit length C_l :
  - Capacitor C in between the upper strip and the ground plane:





$$\Rightarrow C \approx \varepsilon \cdot \frac{L \cdot W}{h}$$

→ Low-pass filter behaviour

  - Capacitor per unit length $$C_l \approx \varepsilon \cdot \frac{W}{h}$$

→ /!\ : the value of the capacitor is underestimated by 20-30% because facing surfaces are larger than $L \cdot W$

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line

- Inductance per unit length L_l

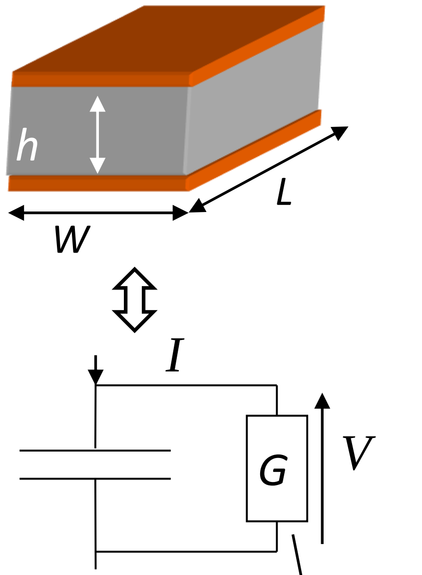
→ **Self-induction effect**

- Corresponds to the phenomenon of propagation: the voltage measured at any point on a Tline is not constant

- Depending on the Tline geometry:
  - Inductance is non-trivial to calculate (application of Ampere's theorem).
  - Typically, the linear inductance of a copper track is ~0.1 to 2 nH/mm.

Appendix: how to obtain the elements of the electrical model?

Example of a microstrip line



- Conductance per unit length G_l
  - Real dielectric material:
    - Electrically insulating and therefore weakly conductive
    - → Leakage currents
    - Conductivity of the dielectric material:

$$\sigma_{diél} = \omega \cdot \varepsilon \cdot tan(\delta)$$

Linked to dielectric losses

$$C_l = \varepsilon \cdot \frac{W}{h}$$

$$G_l = \frac{1}{R_{diel\_l}}$$

- Conductance per unit length: $\boxed{G_l = \omega \cdot C_l \cdot tan(\delta)}$

Appendix 2

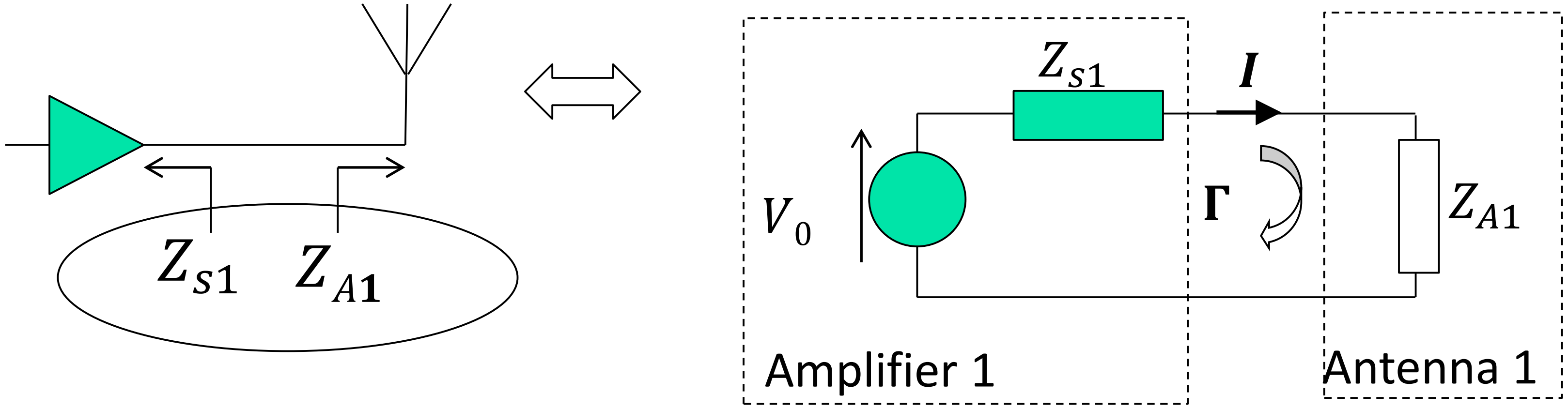
Impedance matching

Appendix: impedance matching

- How to maximize the power transmitted to the load
- Minimize reflections at the interfaces

→ **impedance matching**

Appendix: impedance matching

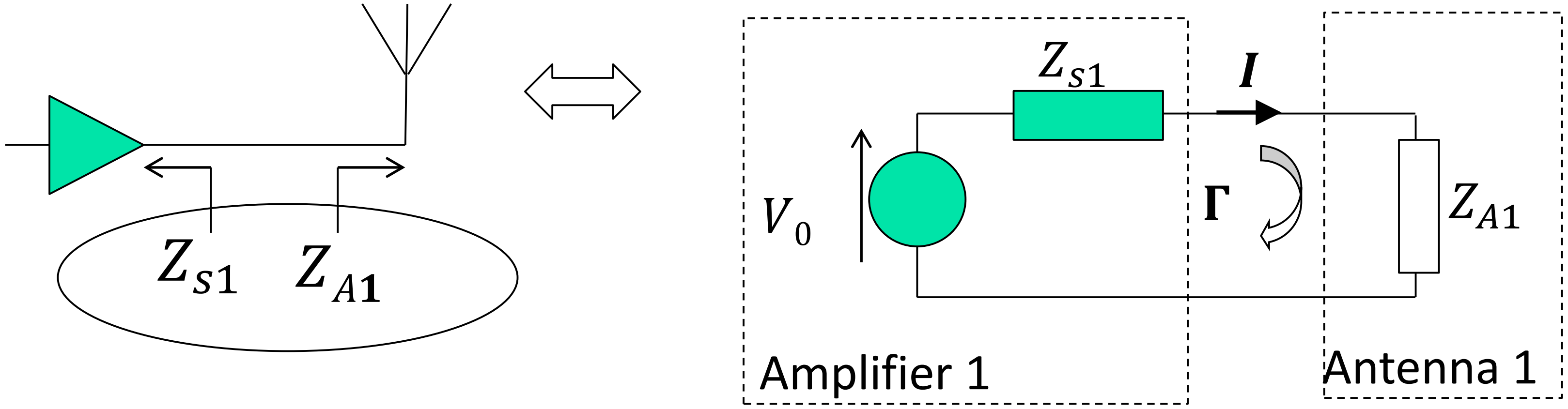


- Express the reflection coefficient Γ $\rightarrow \Gamma = \frac{Z_{A1}-Z_{s1}}{Z_{A1}+Z_{s1}}$

- Calculate the power P_{A1} delivered to the antenna

$$P_{A1} = Z_{A1} \cdot I^2 \qquad \text{with } I = V_0/(Z_{s1} + Z_{A1}) \qquad \Rightarrow P_{A1} = V_0^2 \cdot \frac{Z_{A1}}{(Z_{s1}+Z_{A1})^2}$$

Appendix: impedance matching



- Determine the matching condition (→ maximize the power P_{A1_max} transmitted to the load).
  - The transmitted power P_t to the load is :

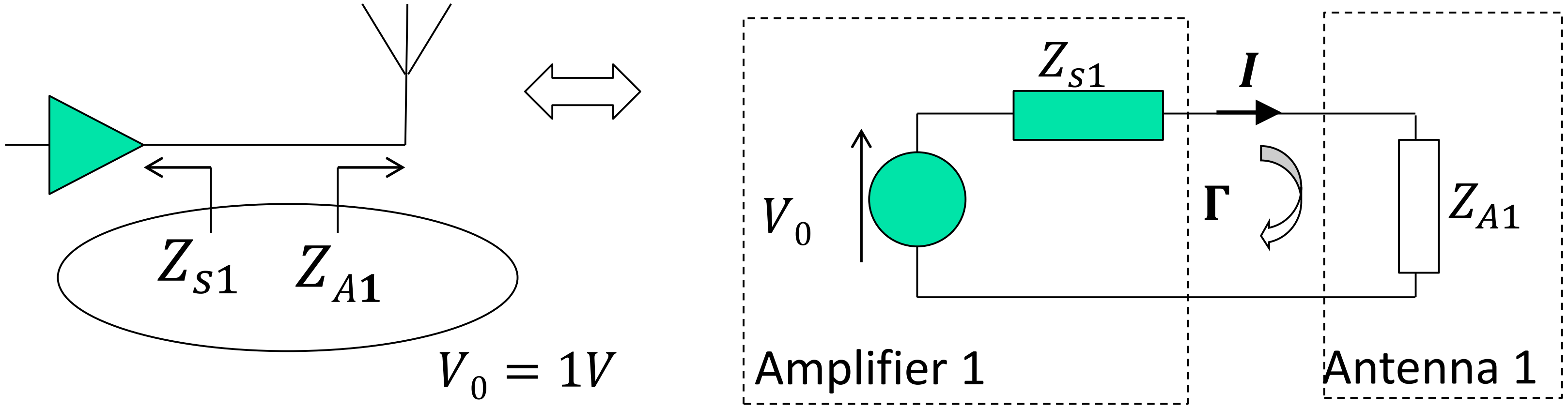
$$\boxed{P_t = (1 - |\Gamma|^2) \cdot P_{in}}$$

To maximize this power → $\Gamma = 0$ (reflection coefficient should be null)

$$\Gamma = \frac{Z_{A1} - Z_{s1}}{Z_{A1} + Z_{s1}} \quad \rightarrow \quad \Gamma = 0 \Leftrightarrow Z_{A1} = Z_{s1}$$

$$P_{A1} = V_0^2 \cdot \frac{Z_{A1}}{(Z_{s1} + Z_{A1})^2} \quad \Rightarrow P_{A1\_max} = \frac{V_0^2}{4Z_{s1}}$$

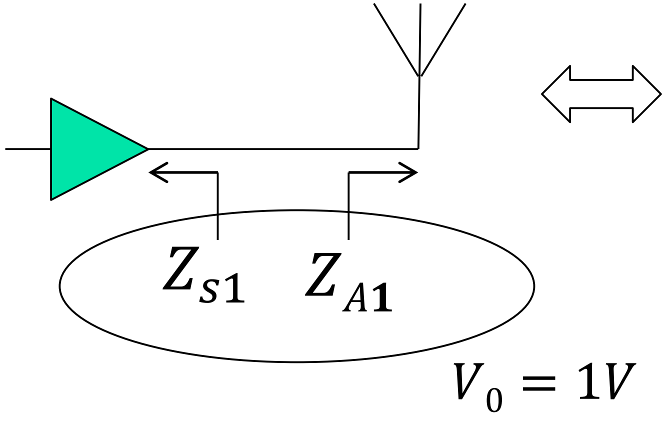
Appendix: impedance matching

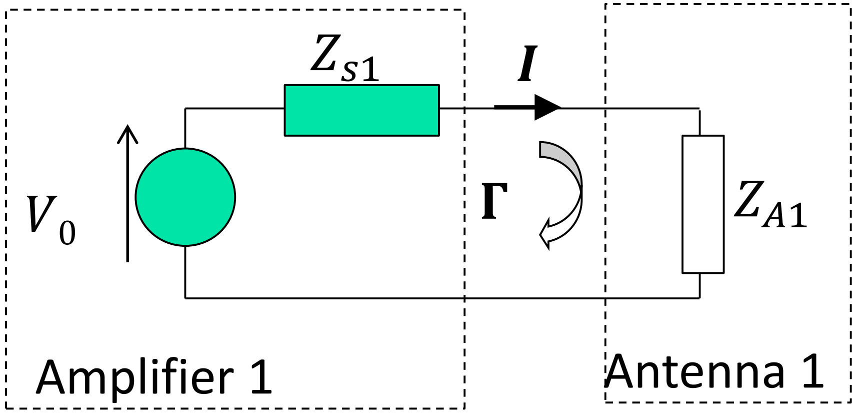


- Calculate the power P_{A1} delivered to the load (antenna) when :
  - Z_{A1}= 50 Ω et Z_{s1} = 50 Ω $\Rightarrow P_{A1\_max} = \frac{V_0^2}{4Z_{s1}} = \frac{1^2}{4 \times 50} = 5\ mW$
  - Z_{A1}= 100 Ω et Z_{s1} = 50 Ω $\Rightarrow P_{A1} = V_0^2 \cdot \frac{Z_{A1}}{(Z_{s1} + Z_{A1})^2} = V_0^2 \cdot \frac{100}{(50+100)^2} = 4{,}44mW$
  - Z_{A1}= 25 Ω et Z_{s1} = 50 Ω $\Rightarrow P_{A1} = V_0^2 \cdot \frac{Z_{A1}}{(Z_{s1} + Z_{A1})^2} = V_0^2 \cdot \frac{25}{(50+25)^2} = 4{,}44mW$

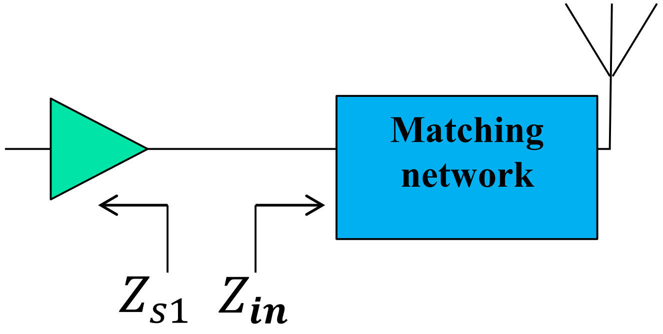
To maximize P_A
$Z_{A1} = Z_{S1}$
(→ $\Gamma = 0$)

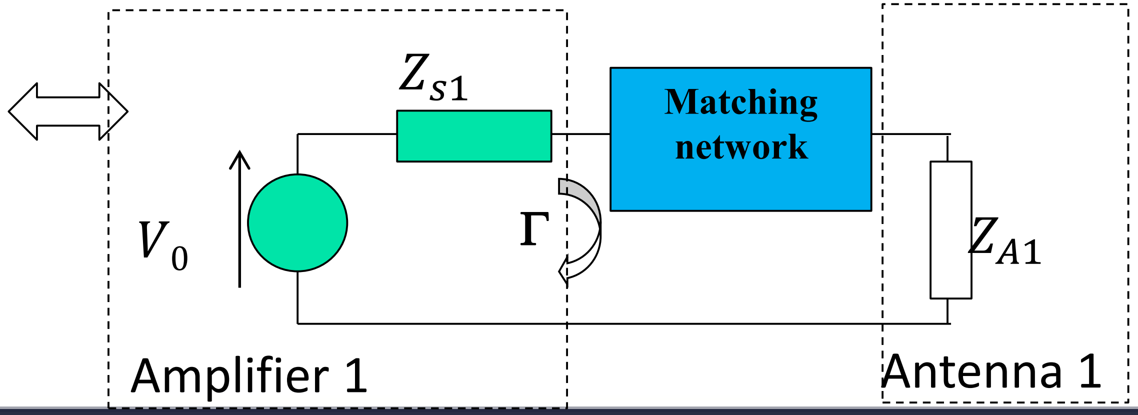
Appendix: impedance matching



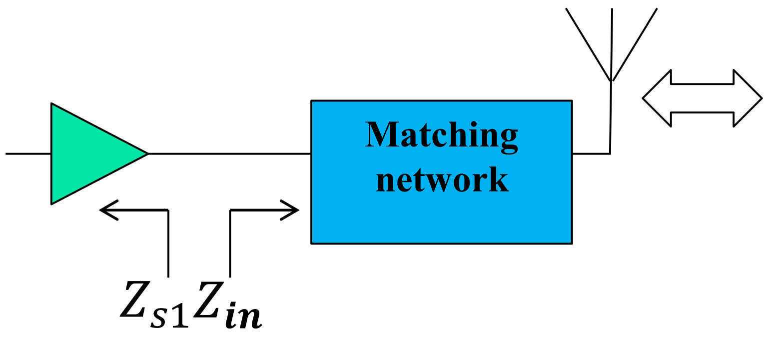


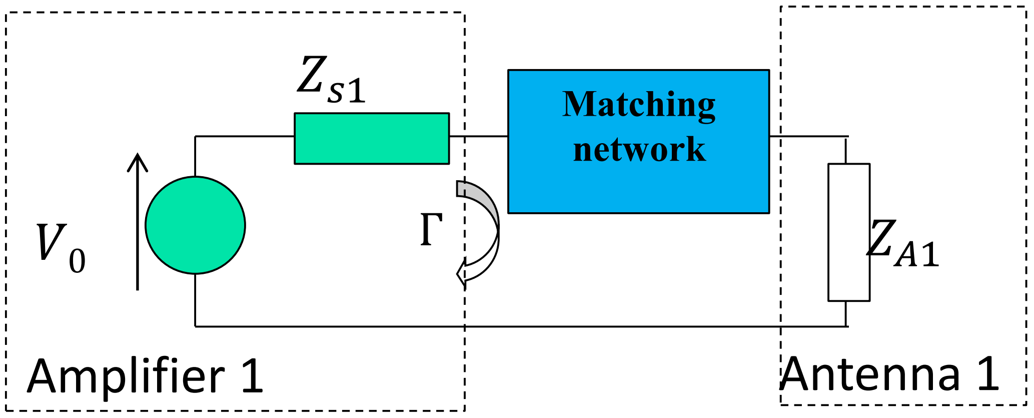
- But, how to maximize the power P_{A1} delivered to the load (antenne) if $Z_{A1} \neq Z_{s1}$: A matching network allows obtaining the matching in btween these two circuits





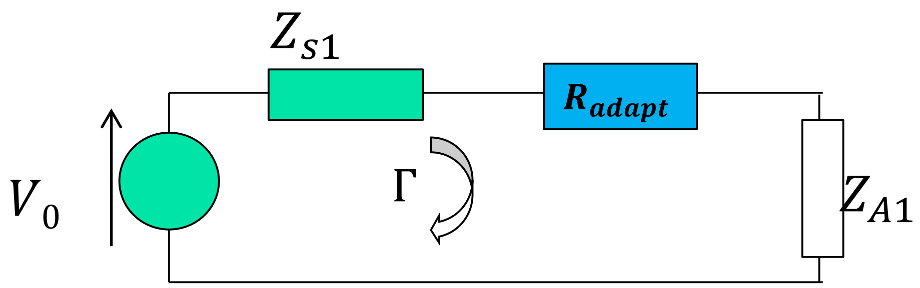
Appendix: impedance matching





- First try:
  - Perhaps could we consider a resistor to match the circuits to each other ? But is it a good solution ?

Example : consider Z_{A1}= 25 Ω et Z_{s1} = 50 Ω

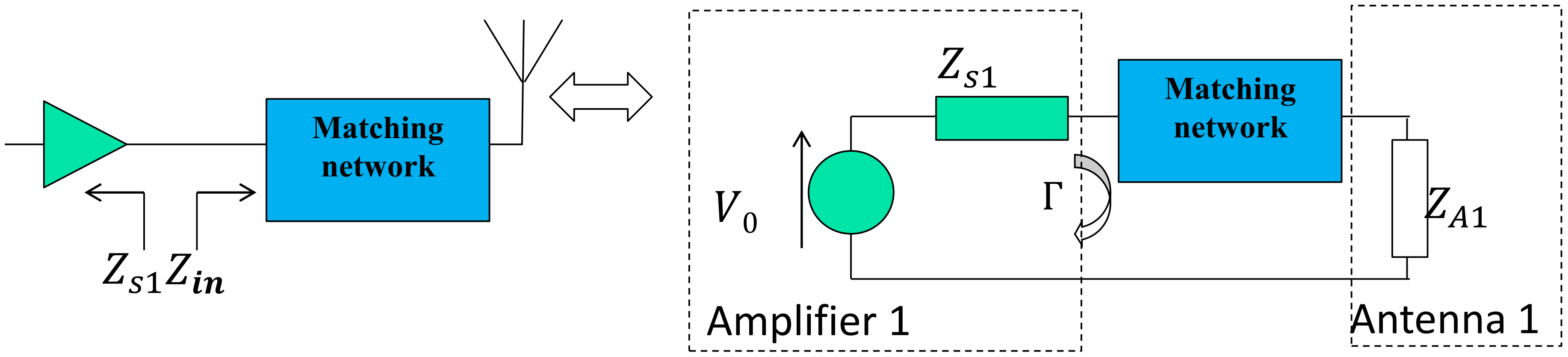


To obtain $\Gamma = 0$ → $R_{adapt} = 25\ \Omega$

BUT, if one calculate the power transmitted to the load:
- only half of the power is transmitted to Z_{A1} (=2,5mW)
- the second half of the power is dissipated in R_{adapt} (=2,5mW)

→ **Thus, using a matching resistor is not a good idea!** ☹

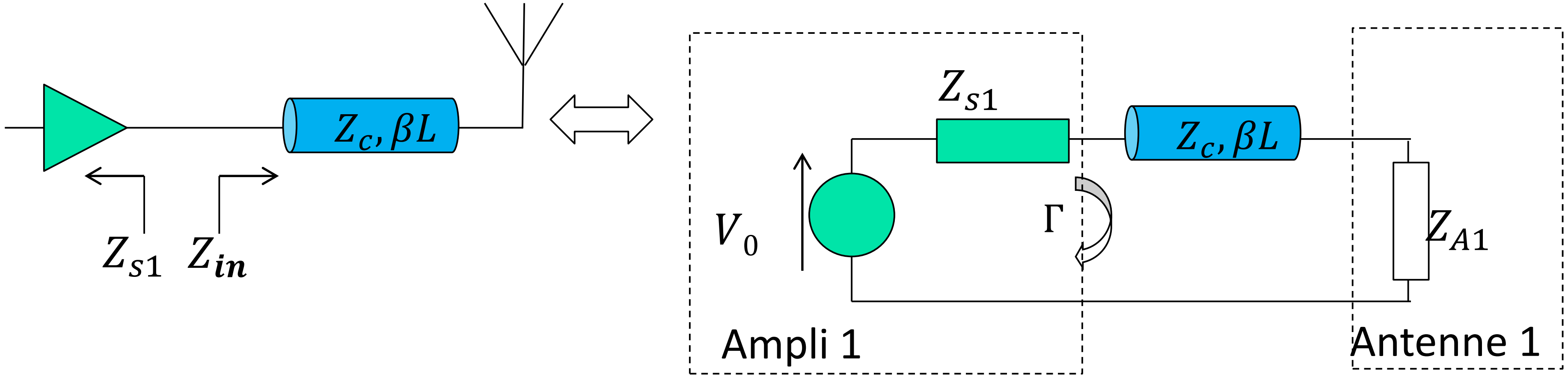
Appendix: impedance matching



- Second try:
  - To avoid this problem of power loss at the output (when considering R_{adapt}), a matching circuit based on inductors and capacitors or on transmission lines could be used

Appendix: impedance matching

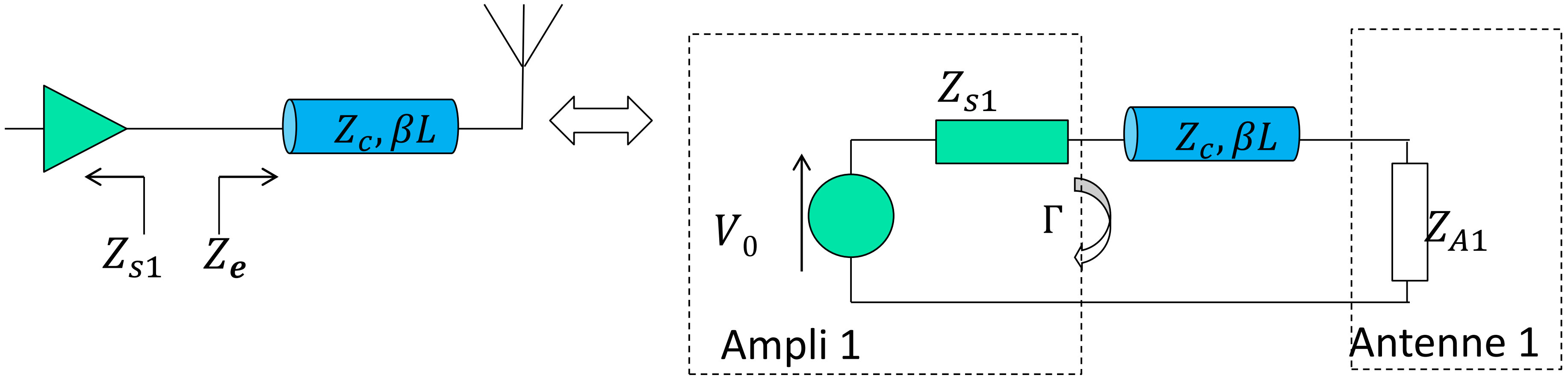
Example considering a transmission line as matching network



- Calculate the input impedance Z_{in} of the circuit constituted of the Tline loaded by the load Z_{A1}.

$$Z_{in} = Z_c \frac{(Z_L + j \cdot Z_c \cdot tan(\beta L))}{(Z_c + j \cdot Z_L \cdot tan(\beta L))} \rightarrow Z_{in} = Z_c \frac{(Z_{A1} + j \cdot Z_c \cdot tan(\beta L))}{(Z_c + j \cdot Z_{A1} \cdot tan(\beta L))}$$

Appendix: impedance matching

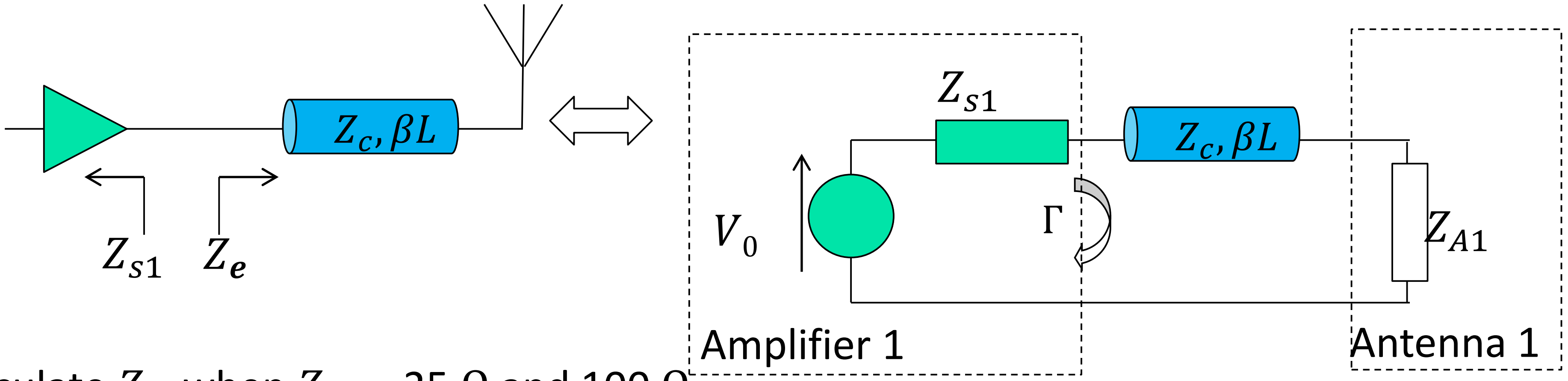


- Show that, if the length $L = \lambda/4$ of the matching Tline, then the input impedance $Z_{in} = \frac{Z_c^2}{Z_{A1}}$

$$L = \lambda/4 \quad \rightarrow \theta = \beta\, L = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \quad \rightarrow tan(\beta L) \rightarrow +\infty$$

$$\rightarrow Z_{in} = \frac{Z_c^2}{Z_{A1}}$$

Appendix: impedance matching



- Calculate Z_c when Z_{A1} = 25 Ω and 100 Ω.

  Z_{in} should be equal to 50 Ω and $Z_{in} = \frac{Z_c^2}{Z_{A1}}$ thus $Z_c = \sqrt{Z_{in} \cdot Z_{A1}}$

  If $Z_{A1} = 25\ \Omega$ → $Z_c = \sqrt{Z_{in} \cdot Z_{A1}} = 35.35\ \Omega$

  If $Z_{A1} = 100\ \Omega$ → $Z_c = \sqrt{Z_{in} \cdot Z_{A1}} = 70.7\ \Omega$

- Considering that $\lambda = 6{,}94\ mm$ at f = 2,45 GHz, calculate the line length L of the matching Tline

  $L = \lambda/4 = 6.94/4 = 17.35mm$