



M1 EECS (Electrical Engineering and Control Systems)

High frequency electronics : Propagation & reflection

Microwave passive circuits

Part 2 : S parameters and Microstrip line

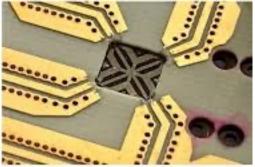
Florence PODEVIN, Emmanuel PISTONO, Jean-Daniel ARNOULD Jean-Marc DUCHAMP 2023-2024



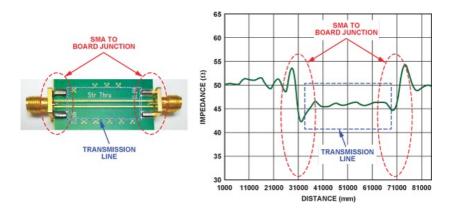
Outline (Part 2) S Parameters and microstrip line

Introduction

- Fields E, H
- Transmission line parameters
- S parameters
 - Introduction & definition
 - Examples quadripoles
- Microstrip line
- Conclusion



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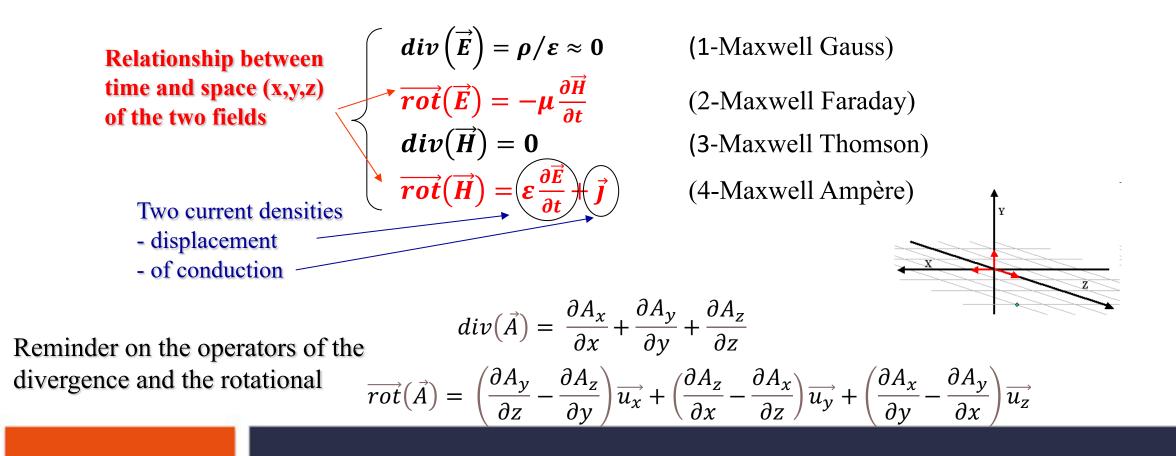




(Part 2) Wireless - Maxwell's equations

The theory of propagation is described by the 4 Maxwell-Lorentz equations.

- These equations describe electromagnetic parameters:
- \vec{E} (electric field V/m), \vec{H} (magnetic field A/m) linked in time and space



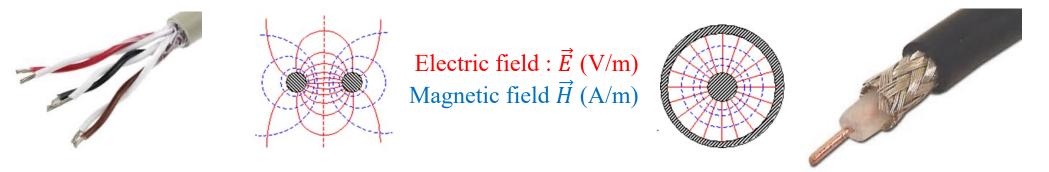
- Consider an electric field $\vec{E}(z,t)$ in vacuum (permittivity ϵ_0 , permeability μ_0) defined by $\vec{E}(z,t) = E_m \sin(\omega t kz) \overrightarrow{e_y}$ in $(0, \overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z})$ (direct orthonormal frame).
 - Represent the field $\vec{E}(z,t)$ at a given instant t=0.
 - Show that the field $\vec{E}(z, t)$ satisfies the Maxwell-Gauss equation.
 - Calculate the rotational coefficient of the field $\vec{E}(z,t)$
 - Determine the expression for the field $\vec{H}(z,t)$ using the Maxwell-Faraday equation -Show the fields $\vec{E}(z,t)$ and $\vec{H}(z,t)$ at a given instant.

- Show that the ratio $|\vec{E}(z,t)|/|\vec{H}(z,t)|$ depends only on the characteristic properties of vacuum

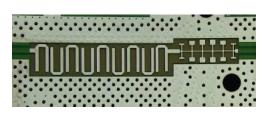


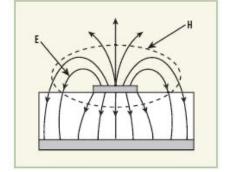
(Part 2) : Transmission line - Fields distribution

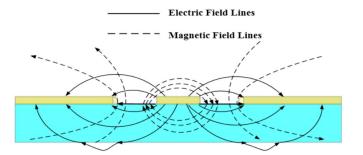
- How to link electric and magnetic field to voltage and current ?
 - Fields distribution in Cables : twisted pair cable and ocaxial cable

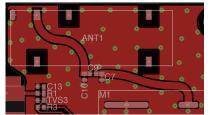


Fields distribution in planar transmission lines : microstrip line and coplanar wave guide







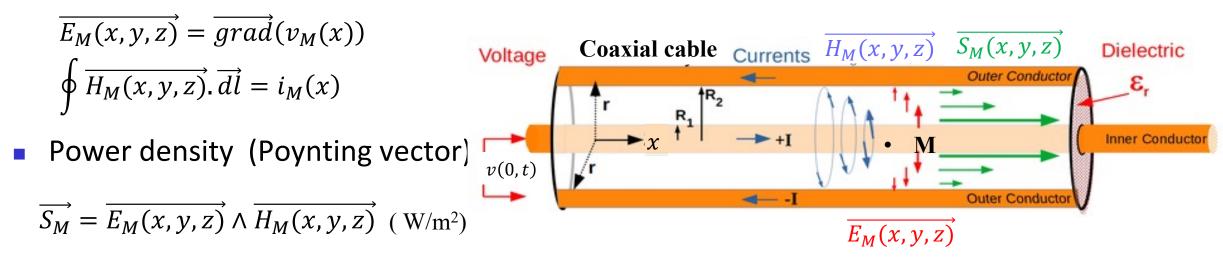


The distribution of the fields \vec{E} and \vec{H} is the same for any section and they are orthogonal



(Part 2) : Transmission line - Voltage & Current

- In case of same distribution of electrical field and magnetic field along the propagation direction x the wave impedance (in Ω) (the ratio $|\vec{E}| / |\vec{H}|$) is constant
- At point M : voltage and curent could expressed as follow

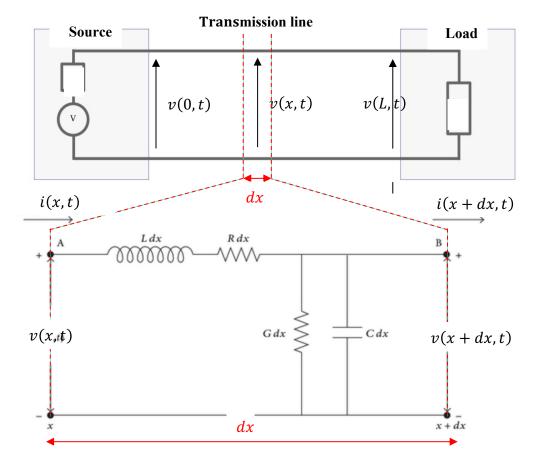


Voltage and Curent in a homogeneous cable or transmission line depends only with the direction of propagation *x* (1D propagation)



(Part 2) : Transmission line - Primary parameters

- Electrical model of a propagation line (telegraph's model)
- Equivalent distributed model for a transmission line segment of length dx (with dx << λ)
- Transmission Line Primary Parameters
 - L (H/m): series linear inductance (magnetic behavior of the transmission line).
 - R (Ω/m) : series linear resistance (conductive losses of the transmission line).
 - C (F/m) : parallel linear capacitance (electrostatic behavior of the transmission line)
 - G (S/m) : parallel linear conductance (dielectric losses of the transmission line)



(Part 2) : Transmission line - Secondary parameters

• Characteristic impedance is the ratio of voltage over current for each wave (progressive and regressive) (linked to ratio $|\vec{E}| / |\vec{H}|$)

$$\overline{Z_{C}} = \frac{\overline{V}_{+}(x,\omega)}{\overline{I}_{+}(x,\omega)} = -\frac{\overline{V}_{-}(x,\omega)}{\overline{I}_{-}(x,\omega)} = \sqrt{\frac{(R+jL\omega)}{(G+jC\omega)}} = \sqrt{\frac{L}{C}} \cdot \sqrt{\frac{(1+R/jL\omega)}{(1+G/jC\omega)}}$$

Transmission Line secondary Parameters (in low-loss condition)

•
$$Z_c(\Omega)$$
 : characteristic impedance $\rightarrow \overline{Z_c} = \sqrt{\frac{L}{c}} \cdot \sqrt{\frac{(1+R/jL\omega)}{(1+G/jC\omega)}} \approx \sqrt{\frac{L}{c}}$

•
$$\beta$$
 (rad/m) : phase constant $\rightarrow \beta = \frac{\omega}{v_{\varphi}} \approx \omega \sqrt{LC}$ or $v_{\varphi} \approx \frac{1}{\sqrt{LC}}$

• α (nepers/m) : attenuation constant $\rightarrow \alpha \approx \frac{1}{2} \frac{R}{Z_c} + \frac{1}{2} GZ_c$

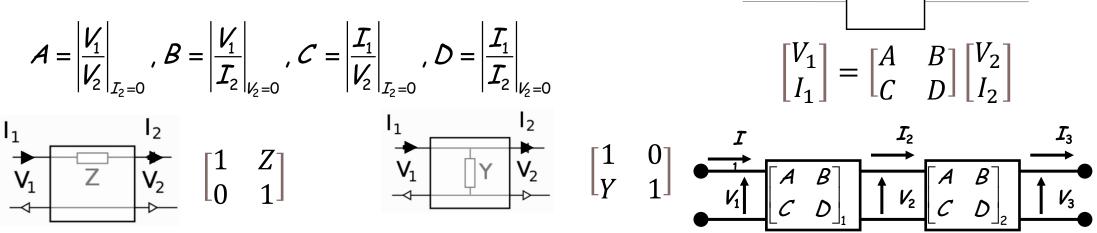
•
$$\alpha_{dB/m} = 20 \cdot Log(e^{\alpha}) = 8,68 \cdot \alpha_{np/m}$$



2-ports

V (Part 2) S parameters- ABCD matrix

- Matrixs are useful to describe the relationship between the output and input parameters or 2 ports network
- For example ABCD matrix or cascaded matrix



• Give $\frac{V_o}{V_i}$ for Telegraph model by cascading ABCD matrix

ABCD matrix based on voltage and current (addition of progressive and regressive waves) are not convient to describe propagation and reflection (steady wave) power would be more appropriated



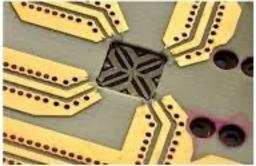
(Part 2) S parameters- ABCD matrix

- Advantages:
 - Very intuitive
 - Describes each access by current voltage
 - Simple calculation for cascaded systems
- Disadvantages:
 - Difficult to measure directly due to reflection and propagation
 - So another matrix must be defined (based on power): the S-matrix which is more easily measured!
 - Matrix ABCD deduced from the measurement by conversion of the measured S-matrix

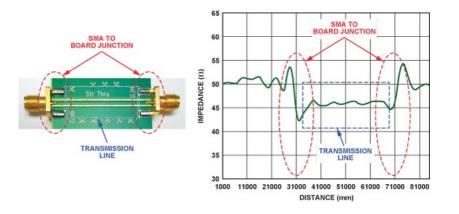
Cutline (Part 2) Microwave : S Parameters and microstrip Time

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(Part 2) S parameters- Waves definition

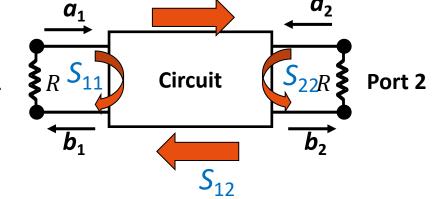
- S-parameters (Scattering Matrix) characterises the circuit or system by providing the transmitted & reflected power waves
 - *a_i* : input wave (square root of the power injected at port *i*)

$$a_1 = \frac{V_1 + Z_c \cdot I_1}{2\sqrt{Z_c}}$$
. $a_2 = \frac{V_2 - Z_c \cdot I_2}{2\sqrt{Z_c}}$ Port 1

b_j : output wave (square root of the power output from port *j*)

$$b_1 = \frac{V_1 - Z_c \cdot I_1}{2\sqrt{Z_c}}$$
 $b_2 = \frac{V_2 + Z_c \cdot I_2}{2\sqrt{Z_c}}$

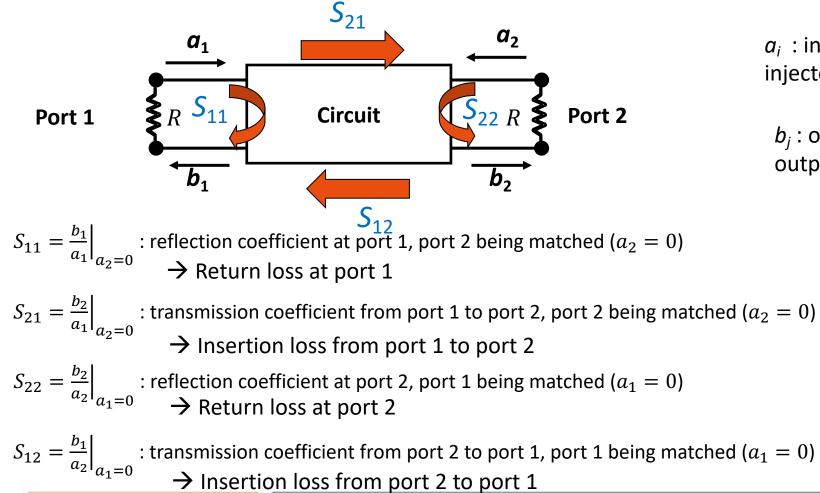
 Interest :Power measurement at each access with well defined load impedances corresponding to practical loads (standard 50 Ohm) allows to bypass the problem of CO & CC*.





(Part 2) S parameters- Scattering matrix definition

 Scattering parameters (or S parameters) : allows a two port network to be described by waves ratio



 a_i : input wave (\rightarrow square root of the power injected at the access *i*)

 b_j : output wave (\rightarrow square root of the power output from the access *j*)

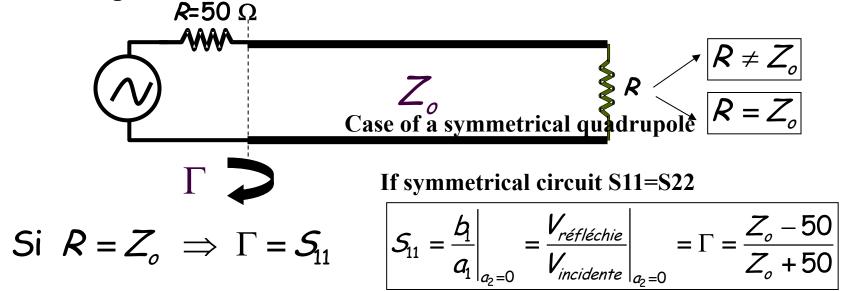
 $b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2$ $b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2$

 $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



V (Part 2) S parameters- Return loss

• Return loss or $S_{11} \rightarrow$ this parameter describe the impedance matching

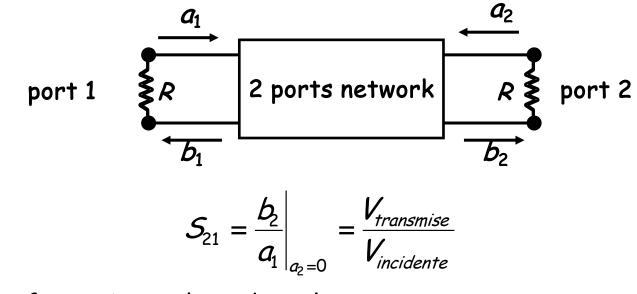


When the load is matched, the measurement of the power reflected back to the source is parameter S_{11} "Return Loss". Case of a symmetrical quadrupole If symmetrical circuit $S_{11}=S_{22}$



χ (Part 2) S parameters- Insertion loss (IL)

- Insertion loss or S_{21}
 - Parameter S_{21} represents the insertion loss or gain of the circuit between the source and the load



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Case of a reciprocal quadrupole :

If passive reciprocal circuit $S_{21} = S_{12}$



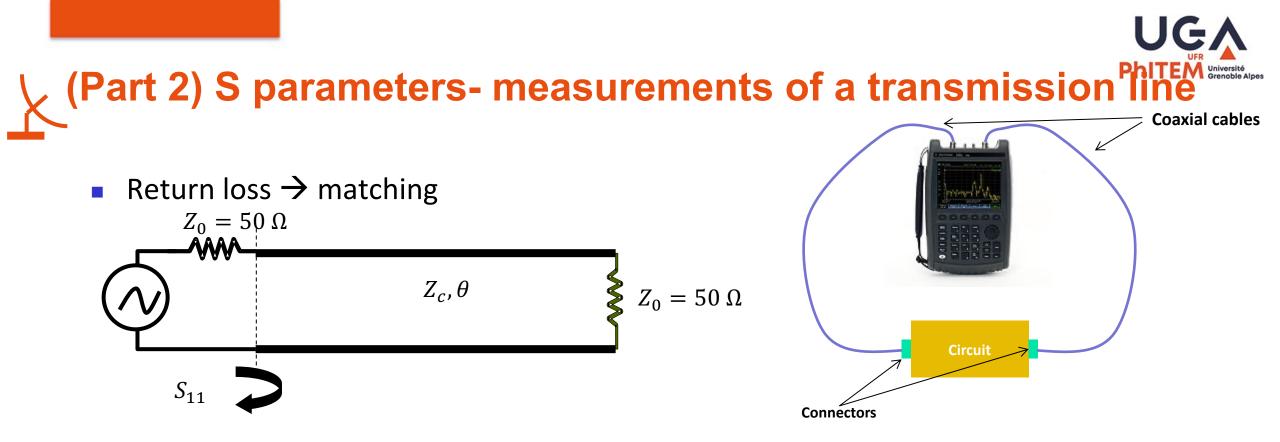
V (Part 2) S parameters- Scattering matrix properties

- Properties :
 - Symetric 2 ports network $\rightarrow S_{11}=S_{22}$
 - Reciprocal 2 ports network $\rightarrow S_{21} = S_{12}$
 - Lossless circuit:
 - The total power leaving the N ports is equal to the total power entering. Therefore, the total power injected at the input of the circuit must be equal to the sum of the outgoing power at the output and the reflected power at the input:

$$\frac{P_R}{P_{inc}} + \frac{P_T}{P_{inc}} = 1 \qquad \Rightarrow \quad |S_{11}|^2 + |S_{21}|^2 = 1$$

- Case of a lossy circuit:
 - The sum of the transmitted and reflected power is not equal to the incident power. The circuit is lossy!
 The difference represents the losses of the circuit, i.e. the power dissipated by it.
 - The losses can be deduced from the S parameters:

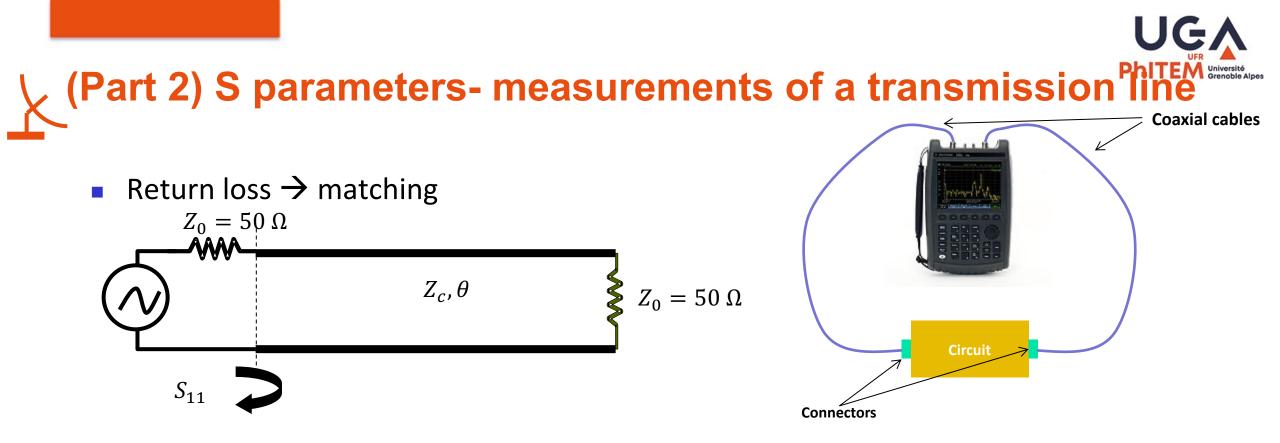
$$\frac{P_{diss}}{P_{inc}} = 1 - (|S_{11}|^2 + |S_{21}|^2)$$



• For a lossless Tline, with a characteristic impedance Z_c and an electrical length $\theta = \beta L_a$ ⊰monstrated for the labwork

$$S_{11} = \frac{j \cdot tan(\theta) \cdot (Z_c^2 - Z_0^2)}{2Z_c Z_0 + j \cdot tan(\theta) \cdot (Z_c^2 + Z_0^2)}$$
 To be defined to the set of the set

• If $Z_c = Z_0 \rightarrow S_{11} = 0 \rightarrow$ the Tline is matched and thus $S_{11} = \Gamma$ because $a_2 = 0$ If $S_{11} = 0 \rightarrow |S_{21}| = \sqrt{1 - |S_{11}|^2} = 1$



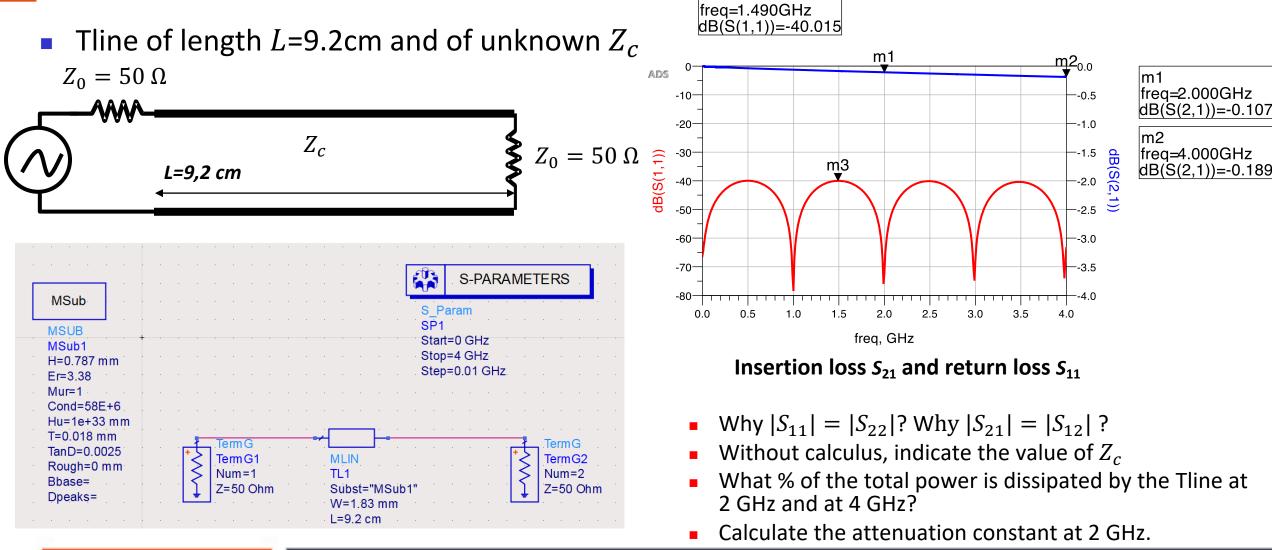
• For a lossless Tline, with a characteristic impedance Z_c and an electrical length $\theta = \beta L$

$$S_{11} = \frac{j \cdot tan(\theta) \cdot (Z_c^2 - Z_0^2)}{2Z_c Z_0 + j \cdot tan(\theta) \cdot (Z_c^2 + Z_0^2)}$$

 $If Z_{c} \neq Z_{0} \rightarrow S_{11} =?$ $If \theta = \frac{\pi}{2}, \frac{3\pi}{2} \dots \rightarrow tan(\theta) \rightarrow +\infty \rightarrow |S_{11}| \approx \frac{(Z_{c}^{2} - Z_{0}^{2})}{(Z_{c}^{2} + Z_{0}^{2})} \text{ and } |S_{21}| = \sqrt{1 - |S_{11}|^{2}}$ $If \theta = 0, \pi, 2\pi \dots \rightarrow tan(\theta) = 0 \rightarrow S_{11} = 0 \text{ and } |S_{21}| = 1$

(Part 2) S parameters- measurements of a transmission line

m3





χ (Part 2) S parameters- S matrix of transmission line

• S-Matrix of a matched transmission line ($S_{11} = S_{22} = 0$) with propagation constant γ and length L is:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-\gamma . L} \\ e^{-\gamma . L} & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-\alpha . L} . e^{-j\beta . L} \\ e^{-\alpha . L} . e^{-j\beta . L} & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

In lossless condition

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-j\beta L} \\ e^{-j\beta L} & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

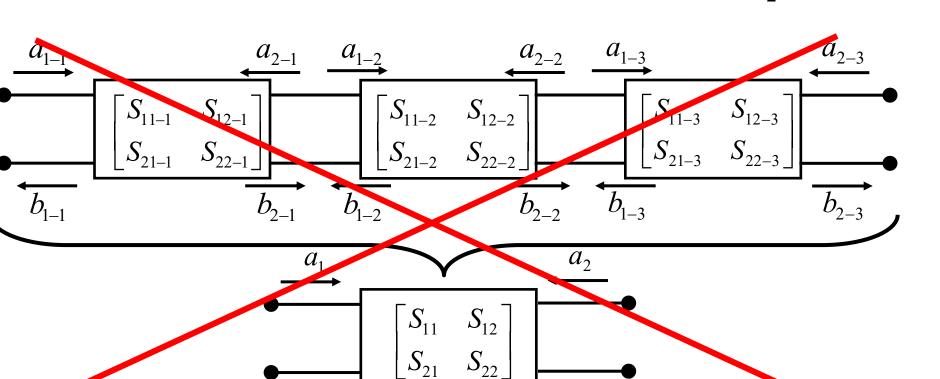


(Part 2) S parameters- S matrix of cascaded circuits

 b_1

Cascaded circuits

Scattering matrix: $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = f\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right)$

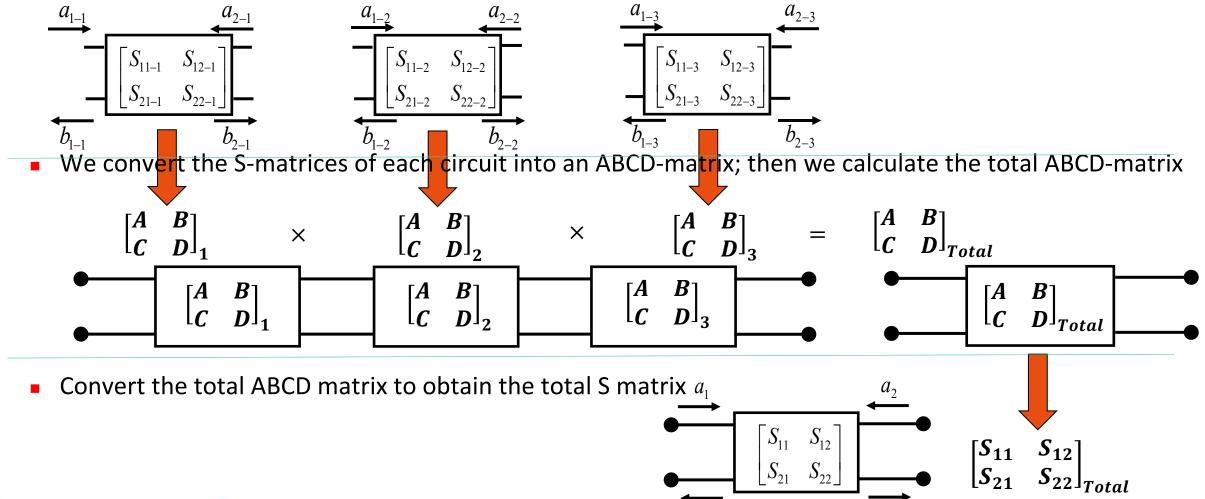


It is incorrect to multiply the S-matrices of each circuit to obtain the S-matrix of the complete system.

 b_{2}

(Part 2) S parameters- S matrix of cascaded circuits

How can we obtain the resulting S matrix of three circuits whose S matrices are known?





V (Part 2) S parameters- Pros and Cons of ABCD Matrix

Conversion formulas: [ABCD] \rightarrow [S] and [S] \rightarrow [ABCD]

$$A = \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}$$
$$B = Z_o \frac{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}{2S_{21}}$$
$$C = \frac{1}{Z_o} \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}}$$
$$D = \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}}$$

$$S_{11} = \frac{A + B / Z_o - CZ_o - D}{A + B / Z_o + CZ_o + D}$$

$$S_{12} = \frac{2(AD - BC)}{A + B / Z_o + CZ_o + D}$$

$$S_{21} = \frac{2}{A + B / Z_o + CZ_o + D}$$

$$S_{22} = \frac{-A + B / Z_o - CZ_o + D}{A + B / Z_o + CZ_o + D}$$

Advantages:

- Very intuitive
- Describes each access by current voltage
- Simple calculation for cascaded systems

Oisadvantages:

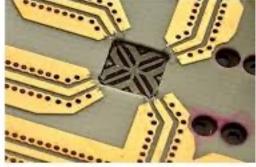
- Difficult to measure → need to consider another matrix: the more easily measurable S-matrix!
- Matrix ABCD deduced from the measurement conversion of the measured S-matrix

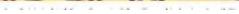


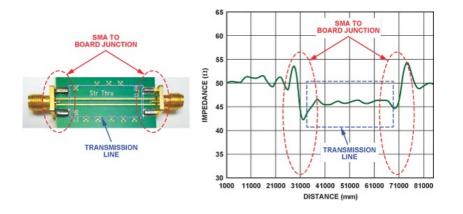
Outline (Part 2) S Parameters and microstrip line

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 - Filter technologies lumped component
 - Transmission line parameters
- Conclusion

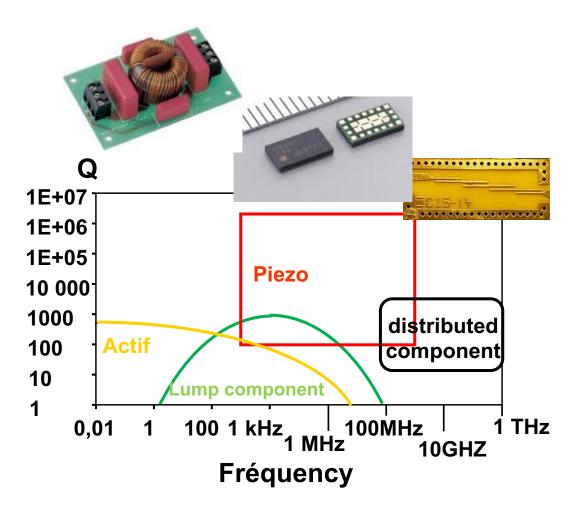






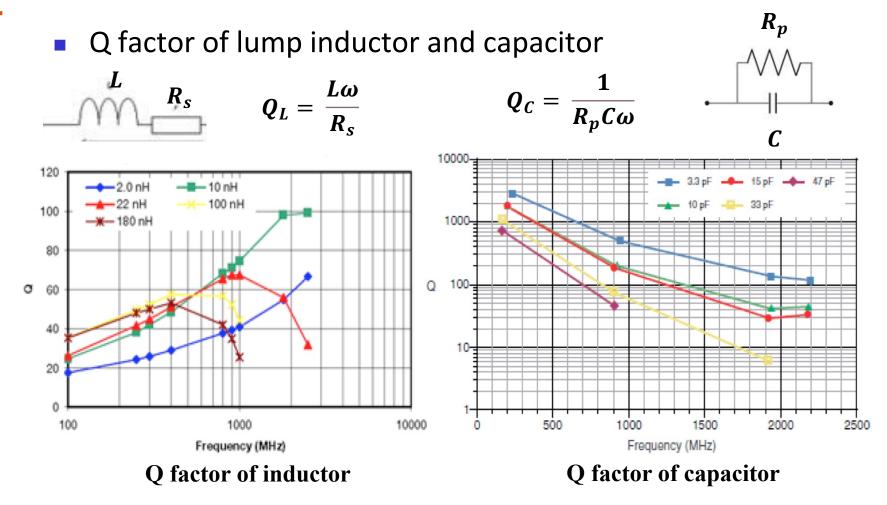
\mathbf{k} (Part2) Introduction : Lumped vs distributed components

- Lump components
 - Capacitance and inductances
- Active filters
 - Operational amplifier and passive R and C components.
- Quartz & piezo electric material
- Distributed components





V (Part2) Introduction : Q factor lumped components



 $Q_C >> Q_L$ for a given frequency and impedance Limiting the use of inductor at RF frequenct. Need to find a over technical solution : distributed component

V (Part2) Introduction : Distributed components with Tlines

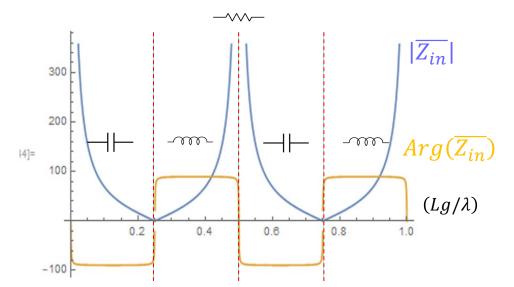
- Tline open stub can synthesize inductor or capacitor
- Several Tline topologies

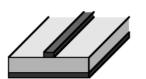




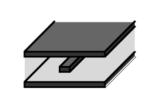
coaxial line

two-wire line (also twisted-pair)





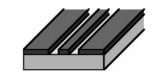
microstrip



stripline



coplanar strip (CPS)



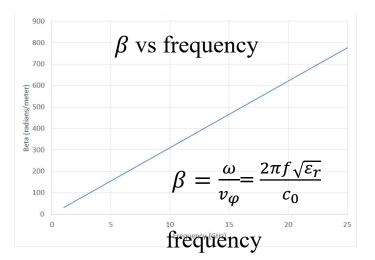
coplanar waveguide (CPW)

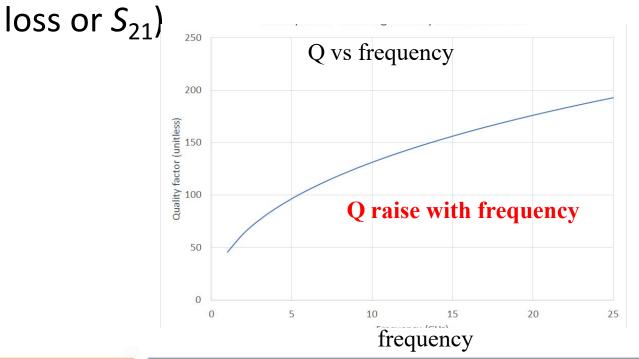
• Which Tline is the most appropriate one ?

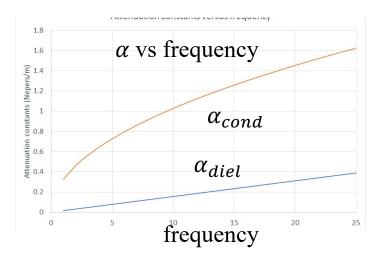


V (Part2) Introduction : Q factor distributed Tline

- The quality factor of Tline : $Q = \frac{1}{2} \frac{\beta}{\alpha}$
- For a transmission line with a length l, we get: $Q = \frac{1}{2} \frac{\beta \cdot l}{\alpha \cdot l} = \frac{1}{2} \frac{\theta}{IL} 8.68$ (IL is Insertion









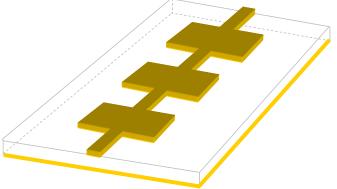
(Part2) Introduction : Examples of transmission lines 100 MHZ 100 642 10 MHZ 10GHz 042 2 GHZ **Two-wire line Twisted 2-wire lines (few meters) Coaxial cable** Coplanar waveguide (CPW) Microstrip line (MS) Rectangular waveguide (RWG) Substrate integrated waveguide (SIW) Metallic vias Metallic layers Substrate

V (Part2) Introduction : Lumped vs distributed components

Problem : Lumped component can not be used above qq GHz: due to propagation problem (to their size) and parasitic element of component reduce the Q factor.

In RF: possibility of making L and C components from transmission line sections!

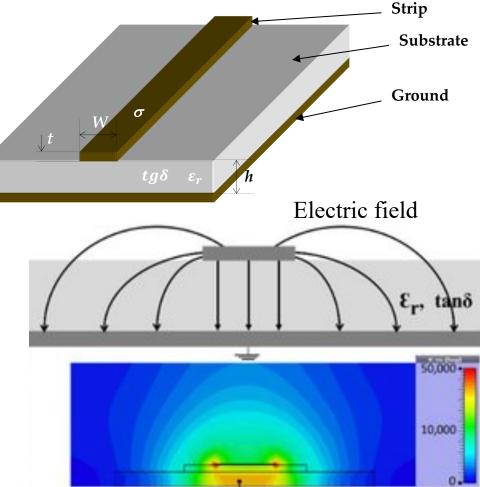
Example of a low-pass filter





V (Part2) 2D Transmission lines topologies : Microstrip

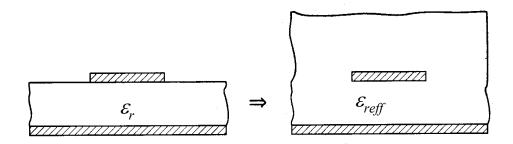
- µstrip is a type of electrical transmission line where a conductor is separated from a ground plane by a dielectric layer (the substrate)
- Microstrip line geometrical parameters are:
 - W: the width of the metallic strip
 - h : the height (thickness) of the substrate
 - *t* : the thickness of the metallic strip
- Microstrip line electrical parameters are:
 - ε_r : the relative dielectric constant of the substrate
 - σ : the conductivity of the metallic strip, and ground plane
 - $tg\delta$: loss tangent or dissipation factor of the substrate





V (Part2) 2D Transmission lines topologies : Microstrip

- Electrical field propagates through an inhomogeneous medium (air and substrate).
- Wave velocity is governed by an ε_{reff} effective relative dielectric constant($1 < \varepsilon_{reff} < \varepsilon_r$)
- Microstrip secondary parameters
 ε_{reff}, Z_c and α can be calculated thanks to
 CAD (Computer Aided Design) formulas.
- In particular, the set of equations proposed by Hammerstad who modifies on Wheeler Z_{micr} are perhaps the most often used

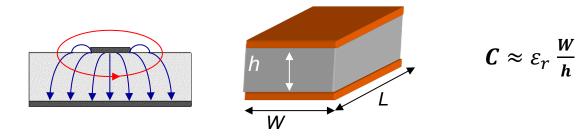


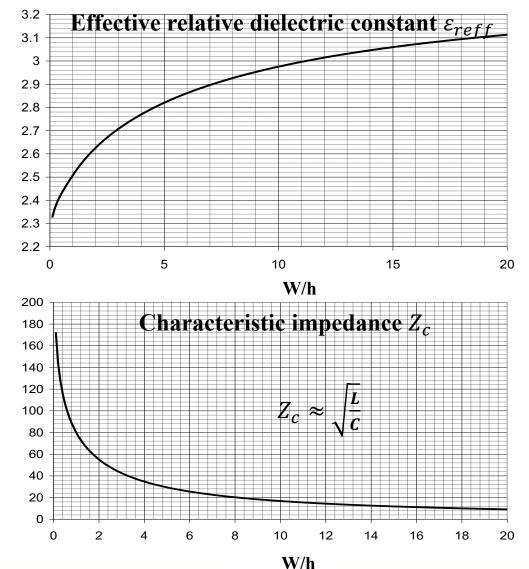
$$arepsilon_{ ext{eff}} = rac{arepsilon_{ ext{r}}+1}{2} + rac{arepsilon_{ ext{r}}-1}{2} \left(rac{1}{\sqrt{1+12(h/w)}}
ight)$$

$$\mathcal{L}_{\mathrm{ostrip}} = egin{cases} rac{Z_0}{2\pi\sqrt{arepsilon_{\mathrm{eff}}}}\lnigg(8rac{h}{w}+rac{w}{4h}igg), & ext{when }rac{w}{h}\leq 1 \ rac{Z_0}{\sqrt{arepsilon_{\mathrm{eff}}}\left[rac{w}{h}+1.393+0.667\lnigg(rac{w}{h}+1.444igg)
ight]}, & ext{when }rac{w}{h}\geq 1 \end{cases}$$

V (Part2) 2D Transmission lines topologies : Microstrip example

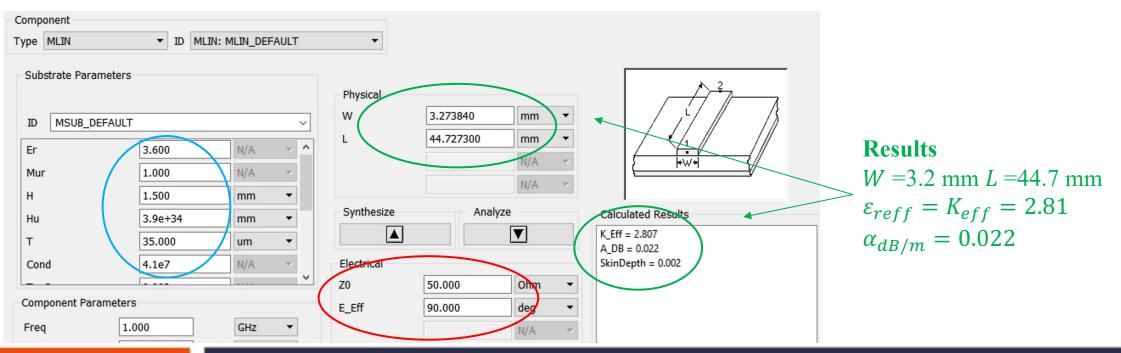
- A microstrip line based on substrate with a thickness h = 1.5 mm, ε_r = 3.36 and dieletric losses tgδ=0.002. The conductor is copper t= 35 μm thick
- Explanation of the shape of the curves Microstrip line equivalent capacitor (primary parameter) could be approximated as a plane capacitor



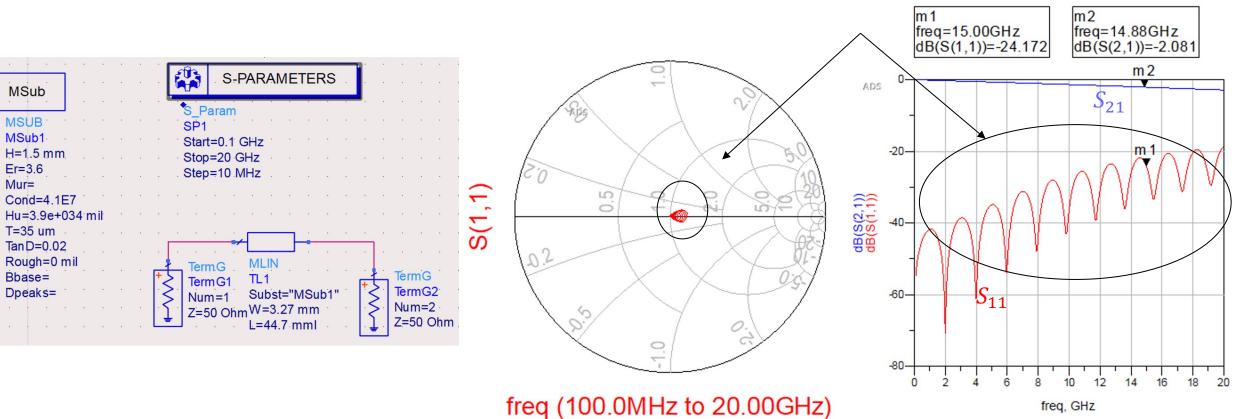


V (Part2) 2D Transmission lines topologies : Microstrip design

- Example of Microstrip line design with LineCal (ADS tool) :
 - $Z_c = 50\Omega$ and $\theta = \beta$. $L = \frac{\pi}{2}$ @ 1GHz
- Substrate
 - geometrical parameters: H = 1.5 mm and $t = 35 \mu m$
 - electrical parameters ε_r =3.6, $tg\delta$ = 0.002 and σ =4.1 10⁷



(Part2) 2D Transmission lines topologies : Microstrip sim



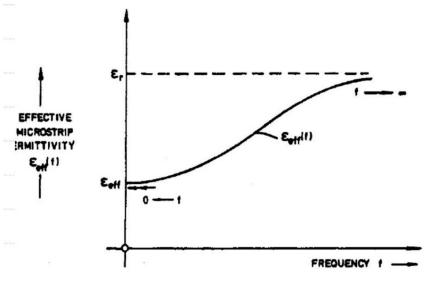
Frequency dispersion of Z_c

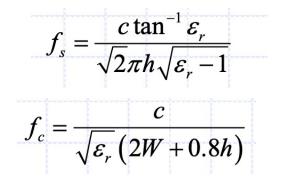
V (Part2) 2D Transmission lines topologies : Microstrip pro

Pro

- planar transmission line simple and very easy to realize
- Cons
 - Complex design secondary parameters equations
 - Dispersion of ε_{reff} and Z_c vs frequency
 - Surface Waves and higher-order modes
 - Coupling between the quasi-TEM mode and surface wave mode become significant when the frequency is above f_s
 - Cutoff frequency f_c of first higher-order modes in a microstrip







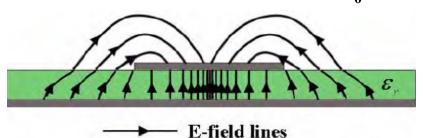
(Part2) 2D Transmission lines topologies : Microstrip

- The non-homogeneity of the microstrip line (propagation in air + dielectric) induces a wave which is not TEM ($\vec{E} \perp \vec{H}$) but quasi-TEM (use of effective relative permittivity)
- First quasi TEM mode EH₀ and higher-order propagation mode EH₁ and EH₂ appear at frequency above cut-off frequency

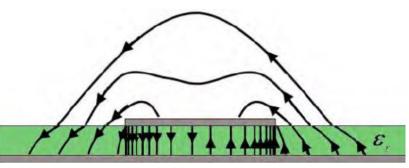
Frequency dispersion 3.9 EH₀ of ε_{reff} EH₁ $\frac{\beta}{k_0}$ 2.0 EH₂ Normalized radiation propagation region constant β/k_0 $\beta = \omega \sqrt{\varepsilon_{reff}} / c_0$ 10 30 20 40 $k_0 = \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ f in GHz



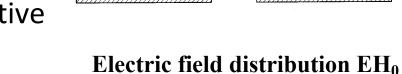
⇒



Electric field distribution EH₁







ε.



(Part 1) Conclusion

- Radio and microwave frequencies (from 100 MHz to 100GHz) are widely used for guided and wireless data transmission to meet the increasing demand for data rate and bandwidth but also for other medical, industrial, military and research applications
- The increase of the frequency induces the necessity to take into account the propagation and reflection phenomena ($L < \frac{\lambda}{20}$ or 5% of λ)
- To describe these phenomena two sets of parameters are available : the primary parameters of a transmission line (RLCG) or the secondary parameters (Zc Vp and Alpha)
- To describe the interactions at the input and output ports, the S-parameters are the most suitable, but they are not cascadable and must be associated with the ABCD-parameters